Quality Model and Artificial Intelligence Base Fuel Ratio Management with Applications to Automotive Engine

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ABSTRACT
In this research, manage the Internal Combustion (IC) engine modeling and a multi-input-multi-output artificial intelligence baseline chattering free sliding mode methodology scheme is developed with guaranteed stability to simultaneously control fuel ratios to desired levels under various air flow disturbances by regulating the mass flow rates of engine PFI and DI injection systems. Modeling of an entire IC engine is a very important and complicated process because engines are nonlinear, multi inputs-multi outputs and time variant. One purpose of accurate modeling is to save development costs of real engines and minimizing the risks of damaging an engine when validating controller designs. Nevertheless, developing a small model, for specific controller design purposes, can be done and then validated on a larger, more complicated model. Analytical dynamic nonlinear modeling of internal combustion engine is carried out using elegant Euler-Lagrange method compromising accuracy and complexity. A baseline estimator with varying parameter gain is designed with guaranteed stability to allow implementation of the proposed state feedback sliding mode methodology into a MATLAB simulation environment, where the sliding mode strategy is implemented into a model engine control module (“software”). To estimate the dynamic model of IC engine fuzzy inference engine is applied to baseline sliding mode methodology. The fuzzy inference baseline sliding methodology performance was compared with a well-tuned baseline multi-loop PID controller through MATLAB simulations and showed improvements, where MATLAB simulations were conducted to validate the feasibility of utilizing the developed controller and state estimator for automotive engines. The proposed tracking method is designed to optimally track the desired FR by minimizing the error between the trapped in-cylinder mass and the product of the desired FR and fuel mass over a given time interval.

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1. INTRODUCTION
Internal combustion engine: Modeling of an entire internal combustion (IC) engine is a very important and complicated process because internal combustion engines are nonlinear, multi inputs-multi outputs (MIMO) and time variant. There have been several engine controller designs over the previous years in which the main goal is to improve the efficiency and exhaust emissions of the automotive engine [1-4]. Specific applications of air to fuel (A/F) ratio control based on observer measurements in the intake manifold were developed by Benninger in 1991 [5]. Another approach was to base the observer on measurements of exhaust gases measured by the oxygen sensor and on the throttle position, which was researched by Onder
mathematical method. Sliding mode methodology is one of the robust and stable methods to tune the fuel ratio in switching sliding mode highly stable method. Artificial intelligence is used to estimate the dynamic model parameters of system. This controller is used to control of highly nonlinear systems, because this type of optimal method is a robust and stable [11-30]. Conversely, pure sliding mode controller is used in many applications; it has two important drawbacks namely; chattering phenomenon, and nonlinear equivalent tracking performance by adjusting controller’s coefficient [51-55].

**Quality Modeling of IC engine:** In developing a valid engine model, the concept of the combustion process, abnormal combustion, and cylinder pressure must be understood. The combustion process is relatively simple and it begins with fuel and air being mixed together in the intake manifold and cylinder. This air-fuel mixture is trapped inside cylinder after the intake valve(s) is closed and then gets compressed. When the air-fuel mixture is compressed it causes the pressure and temperature to increase inside the cylinder. Unlike normal combustion, the cylinder pressure and temperature can rise so rapidly that it can spontaneously ignite the air-fuel mixture causing high frequency cylinder pressure oscillations. These oscillations cause the metal cylinders to produce sharp noises called knock, which it caused to abnormal combustion. The pressure in the cylinder is a very important physical parameter that can be analyzed from the combustion process. After the flame is developed, the cylinder pressure steadily rises, reaches a maximum point after TDC, and finally decreases during the expansion stroke when the cylinder volume increases. Since cylinder pressure is very important to the combustion event and the engine cycle in spark ignition engines, the development of a model that produces the cylinder pressure for each crank angle degree is necessary.

**Result Management based on Nonlinear Controller Methodology to FR Adjust:** A nonlinear robust controller design is major subject in this work. Controller is a device which can sense information from linear or nonlinear system to improve the systems performance [10-13]. The main targets in designing control systems are stability, good disturbance rejection, and small tracking error [14-20]. Several IC engines are controlled by linear methodologies (e.g., Proportional-Derivative (PD) controller, Proportional-Integral (PI) controller or Proportional-Integral-Derivative (PID) controller), but in uncertain dynamic models this technique has limitations [21-45].

**Result Management Based on Sliding Mode Methodology:** Sliding mode methodology (SMM) is a significant nonlinear optimal control fuel ratio methodology under condition of partly uncertain dynamic parameters of system. This controller is used to control of highly nonlinear systems, because this type of optimal method is a robust and stable [11-30]. Conversely, pure sliding mode controller is used in many applications; it has two important drawbacks namely; chattering phenomenon, and nonlinear equivalent dynamic formulation in uncertain dynamic parameter [31-50]. It is possible to solve this problem by combining sliding mode controller and baseline law which this method can helps improve the system’s tracking performance by adjusting controller’s coefficient [51-55].

Result management based on Artificial Intelligence: In recent years, artificial intelligence theory has been used in nonlinear systems. Neural network, fuzzy logic and neuro-fuzzy are synergically combined with nonlinear methodology and used in nonlinear, time variant and uncertain plant (e.g., IC engine). Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory. This controller can be used to control nonlinear, uncertain, and noisy systems. This method is free of some model techniques as in model-based controllers. As mentioned that fuzzy logic application is not only limited to the modelling of nonlinear systems [31-36] but also this method can help engineers to design a model-free controller. The main reasons to use fuzzy logic methodology are able to give approximate recommended solution for uncertain and also certain complicated systems to easy understanding and flexible. Fuzzy logic provides a method to design a model-free controller for nonlinear plant with a set of IF-THEN rules [32]. The applications of artificial intelligence such as neural networks and fuzzy logic in modelling and control are significantly growing especially in recent years.

**Objectives:** Based on previous research IC engine is MIMO, nonlinear and time variant system. One of the most active research areas in field of internal combustion engine (IC engine) is the identification, mathematical modeling and control of the internal combustion engine. Since plant modeling, as accurate as possible, is the fundamental part of any model based control approach, the system is modeled using analytical mathematical method. Sliding mode methodology is one of the robust and stable methods to tune the fuel ratio in IC engine. This method is tuned based on sliding surface slope, to adjust this parameter; based-line methodology is applied to sliding mode method. Artificial intelligence is used to estimate the dynamic model of IC engine. Proposed method is caused to reduce or eliminate the chattering as well as trajectory tracking fuel ratio in switching sliding mode highly stable method.
2. THEOREM

Mathematical Modeling of IC Engine Using Euler Lagrange: In developing a valid engine model, the concept of the combustion process, abnormal combustion and cylinder pressure must be understood. The combustion process is relatively simple and it begins with fuel and air being mixed together in the intake manifold and cylinder. This air-fuel mixture is trapped inside cylinder after the intake valve(s) is closed and then gets compressed [6-9]. When the air-fuel mixture is compressed it causes the pressure and temperature to increase inside the cylinder. In abnormal combustion, the cylinder pressure and temperature can rise so rapidly that it can spontaneously ignite the air-fuel mixture causing high frequency cylinder pressure oscillations. These oscillations cause the metal cylinders to produce sharp noises called knock, which it caused to abnormal combustion. The pressure in the cylinder is a very important physical parameter that can be analyzed from the combustion process. Since cylinder pressure is very important to the combustion event and the engine cycle in spark ignition engines, the development of a model that produces the cylinder pressure for each crank angle degree is necessary.

The dynamic equations of IC engine can be written as:

\[
\begin{bmatrix}
\dot{P}_{FI} \\
\dot{D}_{I}
\end{bmatrix} = \begin{bmatrix}
\dot{M}_{air11} & \dot{M}_{air12} \\
\dot{M}_{air21} & \dot{M}_{air22}
\end{bmatrix} \begin{bmatrix}
\dot{FR} \\
\dot{d}_{i}
\end{bmatrix} + \begin{bmatrix}
P_{motor1} \\
P_{motor2}
\end{bmatrix} \begin{bmatrix}
\dot{FR} \\
\dot{d}_{i}
\end{bmatrix} + \begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix} \times \begin{bmatrix}
\dot{FR}^2 \\
\dot{d}_{i}^2
\end{bmatrix} + \begin{bmatrix}
M_{a1} \\
M_{a2}
\end{bmatrix}
\]

(1)

There for to calculate the fuel ratio and equivalence ratio we can write:

\[
\begin{bmatrix}
\dot{FR} \\
\dot{d}_{i}
\end{bmatrix} = \begin{bmatrix}
\dot{M}_{air11} & \dot{M}_{air12} \\
\dot{M}_{air21} & \dot{M}_{air22}
\end{bmatrix}^{-1} \begin{bmatrix}
\dot{P}_{FI} \\
\dot{D}_{I}
\end{bmatrix} - \begin{bmatrix}
P_{motor1} \\
P_{motor2}
\end{bmatrix} \begin{bmatrix}
\dot{FR} \\
\dot{d}_{i}
\end{bmatrix} + \begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix} \times \begin{bmatrix}
\dot{FR}^2 \\
\dot{d}_{i}^2
\end{bmatrix} + \begin{bmatrix}
M_{a1} \\
M_{a2}
\end{bmatrix}
\]

(2)

To solve \( \dot{M}_{air} \), we can write:

\[
\dot{M}_{air} = \begin{bmatrix}
\dot{M}_{air11} & \dot{M}_{air12} \\
\dot{M}_{air21} & \dot{M}_{air22}
\end{bmatrix}
\]

(3)

Where \( \dot{M}_{air12} = \dot{M}_{air21} \)
Where \( \dot{M}_{air} \) is the ratio of the mass of air.

Matrix \( P_{motor} \) is a \( 1 \times 2 \) matrix:

\[
P_{motor} = \begin{bmatrix}
P_{1} \\
P_{2}
\end{bmatrix}
\]

(4)

Matrix engine angular speed matrix \( (N) \) is a \( 2 \times 2 \) matrix

\[
N = \begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix}
\]

(5)

Where,
Matrix mass of air in cylinder for combustion matrix \( (M_{a}) \) is a \( 1 \times 2 \) matrix.

\[
M_{a} = \begin{bmatrix}
M_{a1} \\
M_{a2}
\end{bmatrix}
\]

(6)

The above target equivalence ratio calculation will be combined with fuel ratio calculation that will be used for controller design purpose.

**Sliding Mode methodology:** Consider a nonlinear single input dynamic system is defined by [10-17]:

\[
x^{(n)} = f(x) + b(x)u
\]

(7)
Where \( u \) is the vector of control input, \( x^{(n)} \) is the \( n^{th} \) derivation of \( x \), \( x = [x, \dot{x}, \ddot{x}, ..., x^{(n-1)}]^T \) is the state vector, \( f(x) \) is unknown or uncertainty, and \( b(x) \) is of known sign function. The main goal to design this controller is train to the desired state; \( x_d = [x_d, \dot{x}_d, \ddot{x}_d, ..., x_d^{(n-1)}]^T \), and trucking error vector is defined by \([18-30]\):

\[
\xi = x - x_d = [\xi, \ldots, \xi^{(n-1)}]^T
\]  

A time-varying sliding surface \( s(x, t) \) in the state space \( \mathbb{R}^n \) is given by \([31-45]\):

\[
s(x, t) = \frac{d}{dt} x^{(n-1)} + \lambda \xi = 0
\]

where \( \lambda \) is the positive constant. To further penalize tracking error, integral part can be used in sliding surface part as follows \([10]\):

\[
s(x, t) = \left( \frac{d}{dt} + \lambda \right)^{n-1} \int_0^t \xi \; dt = 0
\]

The main target in this methodology is kept the sliding surface slope \( s(x, t) \) near to the zero. Therefore, one of the common strategies is to find input \( U \) outside of \( s(x, t) \) \([10]\).

\[
\frac{1}{2} \frac{d}{dt} s^2 (x, t) \leq -\zeta |s(x, t)|
\]

where \( \zeta \) is positive constant.

If \( S(0) > 0 \) \( \rightarrow \frac{d}{dt} S(t) \leq -\zeta \) \( (12) \)

To eliminate the derivative term, it is used an integral term from \( t=0 \) to \( t=t_{reach} \)

\[
\int_{t=0}^{t=reach} \frac{d}{dt} \xi(t) \leq - \int_{t=0}^{t=reach} \eta \rightarrow S(t_{reach}) - S(0) \leq -\zeta (t_{reach} - 0)
\]

Where \( t_{reach} \) is the time that trajectories reach to the sliding surface so, suppose \( S(t_{reach} = 0) \) defined as

\[
0 - S(0) \leq -\eta (t_{reach}) \rightarrow t_{reach} \leq \frac{S(0)}{\zeta}
\]

And

\[
if \; S(0) < 0 \rightarrow 0 - S(0) \leq -\eta (t_{reach}) \rightarrow S(0) \leq -\zeta (t_{reach}) \rightarrow t_{reach} \leq \frac{|S(0)|}{\eta}
\]

Equation (15) guarantees time to reach the sliding surface is smaller than \( \frac{|S(0)|}{\zeta} \) since the trajectories are outside of \( S(t) \).

\[
if \; S_{t_{reach}} = S(0) \rightarrow error(x - x_d) = 0
\]

suppose \( S \) is defined as

\[
s(x, t) = \left( \frac{d}{dt} + \lambda \right) \xi = (\ddot{x} - \dot{x}_d) + \lambda (x - x_d)
\]

The derivation of \( S \), namely, \( \dot{S} \) can be calculated as the following;

\[
\dot{S} = (\ddot{x} - \dot{x}_d) + \lambda (\dot{x} - \dot{x}_d)
\]

suppose the second order system is defined as;

\[
\ddot{x} = f + u \rightarrow \dot{S} = f + U - \dot{x}_d + \lambda (\dot{x} - \dot{x}_d)
\]
Where \( f \) is the dynamic uncertain, and also since \( S = 0 \) and \( \dot{S} = 0 \), to have the best approximation, \( \bar{U} \) is defined as

\[
\bar{U} = -f + \ddot{x}_d - \lambda (\dot{x} - \dot{x}_d) \quad (20)
\]

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

\[
U_{dis} = \bar{U} - K(\dot{x}, t) \cdot \text{sgn}(s) \quad (21)
\]

where the switching function \( \text{sgn}(S) \) is defined as \[11,16]\]

\[
\text{sgn}(s) = \begin{cases} 
1 & s > 0 \\
-1 & s < 0 \\
0 & s = 0 
\end{cases}
\quad (22)
\]

and the \( K(\dot{x}, t) \) is the positive constant. Suppose by (11) the following equation can be written as,

\[
\frac{1}{2} \frac{d}{dt} s^2(x, t) = S \cdot \dot{S} = [f - \tilde{f} - K\text{sgn}(s)] \cdot S = (f - \tilde{f}) \cdot S - K|S| 
\quad (23)
\]

and if the equation (15) instead of (14) the sliding surface can be calculated as

\[
s(x, t) = \left( \frac{d}{dt} + \lambda \right)^2 \left( \int_0^t \ddot{x} \, dt \right) = (\dot{x} - \dot{x}_d) + 2\lambda (\dot{x} - \dot{x}_d) - \lambda^2 (x - x_d) 
\quad (24)
\]

in this method the approximation of \( U \) is computed as \[26]\]

\[
\bar{U} = -\tilde{f} + \ddot{x}_d - 2\lambda (\dot{x} - \dot{x}_d) + \lambda^2 (x - x_d) 
\quad (25)
\]

**Baseline methodology:** The design of a baseline controller to control the fuel ratio was very straightforward. Since there was an output from the fuel ratio model, this means that there would be two inputs into the baseline controller. Similarly, the output of the controller result from the two control inputs of the port fuel injector signal and direct injector signal. In a typical PID controller, the controller corrects the error between the desired output value and the measured value. Since the equivalence ratio and fuel ratio are the two measured signals, two controllers were cascaded together to control the PFI and DI inputs. The first was a PID controller that corrected the error between the desired equivalence ratio and the measured equivalence ratio; while the second was only a proportional integral (PI) controller that corrected the fuel ratio error.

\[
e_1(t) = \alpha_{target}(t) - \alpha_d(t) \quad (26)
\]

\[
e_2(t) = \text{Fuel ratio}_a(t) - \text{Fuel Ratio}_d(t) 
\quad (27)
\]

\[
PFI_{\alpha} = K_{p\alpha} e_1 + K_{v\alpha} \dot{e}_1 + K_{i\alpha} \sum e_1 
\quad (28)
\]

\[
DI_{\alpha} = K_{p\beta} e_1 + K_{v\beta} \dot{e}_1 + K_{i\beta} \sum e_1 
\quad (29)
\]

\[
PFI_{\beta} = (K_{p\gamma} e_2 + K_{i\gamma} \sum e_2) \times PFI_{\alpha} 
\quad (30)
\]

\[
DI_{\beta} = DI_{\alpha} 
\quad (31)
\]

3. **METHODOLOGY**

**Baseline Sliding Mode Controller Methodology:** The design of a baseline controller to control the sliding surface slope was very straightforward. Since there was an output from the sliding surface slope model, this means that there would be two inputs into the baseline controller. Similarly, the output of the controller result from the control input of the sliding surface slope. In a typical PID controller, the controller corrects the error between the desired output value and the measured value. Since the sliding surface slope is
the measured signal, two controllers were cascaded together to control the sliding surface slope. The first was a PID method that corrected the error between the desired FR and the measured FR; while the second was only a proportional integral (PI) method that corrected the sliding surface, error and integral of error.

\[ e(t) = FR_{\text{desired}}(t) - FR_{\text{actual}}(t) \]  

\[ S_a = \lambda_a e + \dot{e} + \left( \frac{\lambda_a}{2} \right)^2 \sum e \]  

\[ S_T = (\lambda_a e + \left( \frac{\lambda_a}{2} \right)^2 \sum e) \times S_a \]  

\[ S_T = (\lambda_a e + \left( \frac{\lambda_a}{2} \right)^2 \sum e) \times (\lambda_a e + \dot{e} + \left( \frac{\lambda_a}{2} \right)^2 \sum e) \]  

**Artificial intelligence (Fuzzy) Based Management Baseline Sliding Mode Methodology:** this part is focused on design SISO fuzzy estimation baseline sliding mode methodology for system’s management based on Lyapunov formulation. The first type of fuzzy systems is given by

\[ f(x) = \sum_{l=1}^{M} \theta^l E^l(x) = \theta^T E(x) \]  

Where

\[ \theta = (\theta^1, ..., \theta^M)^T, E(x) = (E^1(x), ..., E^M(x))^T, \text{ and } E^l(x) = \prod_{i=1}^{n} \mu_{A_i^l}(x_i), \theta^1, ..., \theta^M \] are adjustable parameters in (23). \( \mu_{A_i^l}(x_1), ..., \mu_{A_i^l}(x_n) \) are given membership functions whose parameters will not change over time.

The second type of fuzzy systems is given by

\[ f(x) = \sum_{l=1}^{M} \theta^l \exp \left( \frac{(x_i - \alpha_i^l)^2}{\delta_i^l} \right) \]  

Where \( \theta^l, \alpha_i^l \text{ and } \delta_i^l \) are all adjustable parameters. From the universal approximation theorem, we know that we can find a fuzzy system to estimate any continuous function. For the first type of fuzzy systems, we can only adjust \( \theta^l \) in (23). We define \( f^\ast(x|\theta) \) as the approximator of the real function \( f(x) \).

\[ f^\ast(x|\theta) = \theta^T e(x) \]  

We define \( \theta^\ast \) as the values for the minimum error:

\[ \theta^\ast = \arg \min_{\theta \in \Omega} [\sup_{x \in U} |f^\ast(x|\theta) - f(x)|] \]  

Where \( \Omega \) is a constraint set for \( \theta \). For specific \( x \), \( \sup_{x \in U} |f^\ast(x|\theta^\ast) - f(x)| \) is the minimum approximation error we can get.

We used the first type of fuzzy systems (23) to estimate the nonlinear system (11) the fuzzy formulation can be write as below:

\[ f(x|\theta) = \theta^T e(x) = \sum_{l=1}^{n} \theta^l [\mu_{A_i^l}(x)] \sum_{l=1}^{n} [\mu_{A_i^l}(x)] \]  

Where \( \theta^1, ..., \theta^n \) are adjusted by an adaptation law. The adaptation law is designed to minimize the parameter errors of \( \theta - \theta^\ast \). A MIMO (multi-input multi-output) fuzzy system is designed to compensate the uncertainties of the nonlinear system. The parameters of the fuzzy system are adjusted by adaptation laws. The tracking error and the sliding surface state are defined as:
We define the reference state as

\[ \dot{q}_r = \dot{q} - s = \dot{q}_d - \lambda e \]  

\[ \ddot{q}_r = \ddot{q} - \dot{s} = \ddot{q}_d - \lambda \dot{e} \]

The general MIMO if-then rules are given by

\[ R_l : \text{if } x_1 \text{ is } A^1_1, x_2 \text{ is } A^1_2, \ldots, x_n \text{ is } A^1_n, \ldots, x_1 \text{ is } A^M_1, x_2 \text{ is } A^M_2, \ldots, x_n \text{ is } A^M_n, \]  

then \( y_1 \text{ is } B^1_1, \ldots, y_m \text{ is } B^M_m \) \]

Where \( l = 1, 2, \ldots, M \) are fuzzy if-then rules; \( x = (x_1, \ldots, x_n)^T \) and \( y = (y_1, \ldots, y_m)^T \) are the input and output vectors of the fuzzy system. The MIMO fuzzy system is defined as

\[ f(x) = \Theta^T \varepsilon(x) \]  

Where

\[ \Theta^T = (\theta_1, \ldots, \theta_m)^T = \begin{bmatrix} \theta^1_1, \theta^1_2, \ldots, \theta^1_m \\ \theta^2_1, \theta^2_2, \ldots, \theta^2_m \\ \vdots \\ \theta^m_1, \theta^m_2, \ldots, \theta^m_m \end{bmatrix} \]  

\[ \varepsilon(x) = (\varepsilon^1(x), \ldots, \varepsilon^M(x))^T, \varepsilon^1(x) = \prod_{i=1}^{n} \mu_{A^1_i}(x_i)/\sum_{i=1}^{n} (\prod_{i=1}^{n} \mu_{A^1_i}(x_i)), \]  

and \( \mu_{A^j_i}(x_i) \) is defined in (40). To reduce the number of fuzzy rules, we divide the fuzzy system into three parts:

\[ F^1(q, \dot{q}) = \Theta^T \varepsilon(q, \dot{q}) \]

\[ F^2(q, \ddot{q}) = \Theta^T \varepsilon(q, \ddot{q}) \]

\[ F^3(q, \dddot{q}) = \Theta^T \varepsilon(q, \dddot{q}) \]

The control input is given by

\[ U = \left\{ \left( \begin{array}{c} P_{\text{motor}1} \\ P_{\text{motor}2} \end{array} \right) \left[ FR \quad \bar{a}_{i_{a}} \right] + \left[ \begin{array}{cc} N_{11} & N_{12} \\ N_{21} & N_{22} \end{array} \right] \times \left[ \begin{array}{c} FR_{a} \\ \bar{a}_{i_{a}} \end{array} \right]^2 + \left[ \begin{array}{c} M_{a_{1}} \\ M_{a_{2}} \end{array} \right] \right\} + F^1(q, \dot{q}) + F^2(q, \ddot{q}) + F^3(q, \dddot{q}) - K_d s - W \text{sgn}(s) \]

\[ W = \text{diag}[W_1, \ldots, W_m] \]  

and \( W_1, \ldots, W_m \) are positive constants. The adaptation law is given by

\[ \dot{\theta}_j^1 = -\Gamma_1 j s_f \varepsilon(q, \dot{q}) \]

\[ \dot{\theta}_j^2 = -\Gamma_2 j s_f \varepsilon(q, \ddot{q}) \]

\[ \dot{\theta}_j^3 = -\Gamma_3 j \varepsilon(q, \dddot{q}) \]

Where \( j = 1, \ldots, m \) and \( \Gamma_1 j - \Gamma_3 j \) are positive diagonal matrices.
The Lyapunov function candidate is presented as
\[
V = \frac{1}{2} s^T Ms + \frac{1}{2} \sum_{j=1}^{m} \frac{1}{r_{1j}} \theta_j^T \theta_j + \frac{1}{2} \sum_{j=1}^{m} \frac{1}{r_{2j}} \theta_j^2 + \frac{1}{2} \sum_{j=1}^{m} \frac{1}{r_{3j}} \theta_j^3
\]  
(53)

Where \( \theta_j^0 = \theta_j^+ - \theta_j^−, \theta_j^2 = \theta_j^2 - \theta_j^2 \) and \( \theta_j^3 = \theta_j^3 - \theta_j^3 \) we define
\[
F(q, \dot{q}, \ddot{q}, \tilde{q}) = F^1(q, \dot{q}) + F^2(q, \ddot{q}) + F^3(q, \tilde{q})
\]  
(54)

From (29) and (30), we get
\[
M(q) \ddot{q} + P_m(\theta) + P_{\text{net}}(\theta) = M' \ddot{q}_r + P_m(\theta) + \ddot{F}(q, \dot{q}, \ddot{q}, \tilde{q}) - K_D s - Wsgn(s)
\]  
(55)

Since \( \ddot{q}_r = \ddot{q} - s \) and \( \ddot{q}_r = \ddot{q} - s \), we get
\[
M \dot{s} + (P_m(\theta) + P_{\text{net}}(\theta) + K_D) s + Wsgn(s) = -\Delta F + F(q, \dot{q}, \ddot{q})
\]  
(56)

Then \( M \dot{s} + P_m(\theta) + P_{\text{net}}(\theta) s \) can be written as
\[
M \dot{s} + P_m(\theta) + P_{\text{net}}(\theta) s = -\Delta F + F(q, \dot{q}, \ddot{q}) - K_D s - Wsgn(s)
\]  
(57)

Where \( \Delta F = \ddot{q} \ddot{q} + P_m(\theta) + P_{\text{net}}(\theta), \ddot{q} = M - M^+, \ddot{q}_1 = P_m(\theta) + P_{\text{net}}(\theta) - P_m(\theta) + P_{\text{net}}(\theta) \)

The derivative of \( V \) is
\[
\dot{V} = s^T M \dot{s} + \frac{1}{2} s^T Ms + \sum_{j=1}^{m} \frac{1}{r_{1j}} \theta_j^T \dot{\theta}_j + \sum_{j=1}^{m} \frac{1}{r_{2j}} \theta_j^2 + \sum_{j=1}^{m} \frac{1}{r_{3j}} \theta_j^3
\]  
(58)

We know that \( s^T M \dot{s} + \frac{1}{2} s^T Ms = s^T (M \dot{s} + P_m(\theta) + P_{\text{net}}(\theta) s) \) from (45). Then
\[
\dot{V} = -s^T[-K_D s + Wsgn(s) + \Delta F - F(q, \dot{q}, \ddot{q})] + \sum_{j=1}^{m} \frac{1}{r_{1j}} \theta_j^T \dot{\theta}_j + \sum_{j=1}^{m} \frac{1}{r_{2j}} \theta_j^2 + \sum_{j=1}^{m} \frac{1}{r_{3j}} \theta_j^3
\]  
(59)

We define the minimum approximation error as
\[
\omega = \Delta F - [F^1(q, \dot{q} | \Theta^1) + F^2(q, \ddot{q} | \Theta^2) + F^3(q, \tilde{q} | \Theta^3)]
\]  
(60)

We plug (49) into (50)
\[
\dot{V} = -s^T[-K_D s + Wsgn(s) + \Delta F - F(q, \dot{q}, \ddot{q}, \tilde{q})] + \sum_{j=1}^{m} \frac{1}{r_{1j}} \theta_j^T \dot{\theta}_j + \sum_{j=1}^{m} \frac{1}{r_{2j}} \theta_j^2 + \sum_{j=1}^{m} \frac{1}{r_{3j}} \theta_j^3
\]  
(59)

\[
\sum_{j=1}^{m} \frac{1}{r_{2j}} \theta_j^2 + \sum_{j=1}^{m} \frac{1}{r_{3j}} \theta_j^3 = -s^T[\ddot{q} \ddot{q} + Wsgn(s) + \omega + F^1(q, \dot{q} | \Theta^1) + F^2(q, \ddot{q} | \Theta^2) + F^3(q, \tilde{q} | \Theta^3) - F^1(q, \dot{q}) + F^2(q, \ddot{q}) + F^3(q, \tilde{q})] + \sum_{j=1}^{m} \frac{1}{r_{1j}} \theta_j^T \dot{\theta}_j + \sum_{j=1}^{m} \frac{1}{r_{2j}} \theta_j^2 + \sum_{j=1}^{m} \frac{1}{r_{3j}} \theta_j^3
\]

\[
\sum_{j=1}^{m} \frac{1}{r_{2j}} \theta_j^2 + \sum_{j=1}^{m} \frac{1}{r_{3j}} \theta_j^3 = -s^T[\ddot{q} \ddot{q} + Wsgn(s) + \omega - \sum_{j=1}^{m} \frac{1}{r_{1j}} s_j \theta_j^T \epsilon(q, \dot{q}) - \sum_{j=1}^{m} \frac{1}{r_{1j}} s_j \theta_j^T \epsilon(q, \ddot{q}) - \sum_{j=1}^{m} \frac{1}{r_{1j}} s_j \theta_j^T \epsilon(q, \tilde{q}) + \sum_{j=1}^{m} \frac{1}{r_{1j}} \theta_j^T \dot{\theta}_j + \sum_{j=1}^{m} \frac{1}{r_{2j}} \theta_j^2 + \sum_{j=1}^{m} \frac{1}{r_{3j}} \theta_j^3]
\]
$$\begin{align*}
&= -s^T K_D s - s^T W \text{sgn}(s) - s^T \omega - \\
&\sum_{j=1}^m \phi_j^T (s_j \epsilon(q, \dot{q}) - \frac{1}{\tau_j} \dot{\phi}_j) - \\
&\sum_{j=1}^m \phi_j^T (s_j \epsilon(q, \ddot{q}) - \frac{1}{r_{3j}} \ddot{\phi}_j) - \\
&\sum_{j=1}^m \phi_j^T (s_j \epsilon(q, \dddot{q}) - \frac{1}{r_{3j}} \dddot{\phi}_j)
\end{align*}$$

Then $V$ becomes

$$\dot{V} = -s^T K_D s - s^T W \text{sgn}(s) - s^T \omega$$

$$\begin{align*}
&= -\sum_{j=1}^m (s_j^2 K_{Dj} + W_j |s_j| + s_j \omega_j) \\
&= -\sum_{j=1}^m [s_j (s_j K_{Dj} + \omega_j) + W_j |s_j|]
\end{align*}$$

Since $\omega_j$ can be as small as possible, we can find $K_{Dj}$ that $|s_j^2 K_{Dj}| > |\omega_j|(s_j \neq 0)$.

Therefore, we can get $s_j (s_j K_{Dj} + \omega_j) > 0$ for $s_j \neq 0$ and $\dot{V} < 0 (s \neq 0)$. Figure 3 is shown the fuzzy estimator variable structure.

4. RESULTS and DISCUSSION

This section is focused on comparing between baseline methodology to FR optimization (BLO) and artificial intelligence (fuzzy) baseline Sliding Mode methodology (FBSMM). These controllers were tested by MATLAB/SIMULINK environment.

**FR tracking Managing:** Based on equations in sliding mode methodology; this method performance is depended on the gain updating factor ($K$) and sliding surface slope coefficient ($\lambda$). These two coefficients are computed by gradient descent optimization in pure SMM. After this process the main important challenge is, optimizer is depending on sliding surface slope coefficients so baseline method is apply. Figure 1 shows FR tracking performance in BLO and FBSMM without torque load disturbance.

![Figure 1. BSMM and BLO for adjust FR without torque load disturbance](image_url)

Based on Figure 1 it is observed that, BLO has oscillation in this nonlinear system, caused to instability in tuning the FR but FBSMM has steady in response. BLO’s overshoot is 1.3% but BSMM’s overshoot is 0%.

**Managing the disturbance rejection:** Figures 2 and 3 are shows the power disturbance elimination in BLO and FBSMM with torque load disturbance. The manage disturbance rejection is used to test the robustness comparisons of these two methodologies. A variable limited torque load with predefined of 10% and 20% the power of input signal value is applied to this system.
Based on Figure 2; by comparing FR trajectory with 10% torque load disturbance of relative to the input signal amplitude in BSMM and BLO, BSMM’s overshoot about (0%) is lower than BLO’s (2.3%). Based on Figure 2, it is observed that FBSMM’s performance is better than BLO and it also can eliminate the chattering in presence of 10% disturbance.

Based on Figure 3; by comparing FR trajectory with 20% disturbance of relative to the input signal amplitude in FBSMM and BLO, FBSMM’s overshoot about (0.32%) is lower than BLO’s (3.11%). Based on Figure 3 it is observed that, these two methodologies have oscillation in presence of 20% torque load uncertainty but FBSMM is more stable and more robust than BLO.

5. CONCLUSION

Refer to this research, IC engine modeling based on Lagrange and an artificial intelligence based manage baseline sliding mode methodology is proposed for tuning the FR in internal combustion engine based on nonlinear methodology. Manage the stability and convergence of the fuzzy baseline sliding mode controller based on switching function is guarantee and proved by the LYAPUNOV method. The simulation results exhibit that the fuzzy baseline sliding mode methodology tuning the FR very well in various situations. The FBSMM gives significant steady state error managing performance when compared to BLO. The main goal in this research were modeling and manage or eliminate the chattering that proposed methodology reach to this objective by adding baseline and fuzzy method to high impact robust and stable nonlinear based methodology. The second main goal in this research was reduce dependence on sliding surface slope which baseline methodology does this objective in this research. When applied 20% torque load disturbances in FBSMM the RMS error increased approximately 0.1% (percent of increase the BSMM RMS error = \( \frac{10^{-5}}{10^{-6}} = 0.1\% \)) that in BLO the RMS error increased approximately 8% (percent of increase the BLO RMS error = \( \frac{10^{-6}}{10^{-6}} = 8\% \)).

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REFERENCES


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