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Designing Observer Based Variable Structure Controller for Large Scale Nonlinear Systems

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ABSTRACT

Designing observer based decentralized fuzzy adaptive controller is discussed for a class of large scale non-canonical nonlinear systems with unknown functions of the subsystems in this paper. The On-line adaptation of the controller and the observer parameters, boundedness of the output and the observer errors, robustness against external disturbance are the advantages of the proposed method. The simulation results show the promising performance of the proposed method.

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1. INTRODUCTION

As a result of both tunable structure of the FAC and using the experts' knowledge in controller design procedure, fuzzy adaptive controller (FAC) attracted many researchers to develop appropriate controllers for nonlinear systems especially for large scale systems (LSS).

Nowadays, FAC has been fully studied. Initially, the Takagi-Sugeno (TS) fuzzy systems have been used to model nonlinear systems and then TS based controllers have been designed with guaranteed stability [1]. Modeling affine nonlinear systems and designing stable TS fuzzy based controllers have been employed in [2]. Designing of the sliding mode fuzzy adaptive controller for a class of multivariable TS fuzzy systems were presented in [3].

The linguistic fuzzy systems have also been used to design controllers for nonlinear systems.

[4] has considered linguistic fuzzy systems to design stable adaptive controller for affine systems based on feedback linearization. Stable FAC based on sliding mode was designed for affine systems in [5]. Designing FAC for affine chaotic systems was presented in [6]. Designing stable FAC and linear observers for class of affine nonlinear systems were discussed in [7]. The output feedback FAC for class of affine nonlinear MIMO systems was suggested in [8]. A robust adaptive fuzzy controller, based on a linear state observer, for a class of affine nonlinear systems has been presented in [9]. In [10], direct and indirect adaptive output-feedback fuzzy decentralized controllers for a class of large-scale affine nonlinear systems have been developed based on linear observer. [11] presented fuzzy adaptive controller for a class of affine nonlinear systems. This method guarantied ultimately boundedness of tracking error. A direct adaptive fuzzy controller for a non-minimum phase two-axis inverted pendulum servomechanism has been presented based on real-time stabilization in [12]. The main drawbacks of these papers are those restricted conditions imposed on the system dynamics. For example, it is assumed the control gain is bounded to some known functions or constant values.

[13] developed stable FAC for class of non-affine nonlinear systems. The main limitation of these methods is that convergence of tracking errors to zero was not guaranteed. [14, 15] proposed a decentralized fuzzy model reference state tracking controller for a class of canonical nonlinear large scale system. The main limitations of these references are both considering the interaction as a bounded disturbance and availability of all states.

Compared with the previous studies which mainly concentrated on observer-based affine SISO systems and observer based affine large scale systems, the proposed method is an observer based large scale non-canonical nonlinear systems.

The remainder of the paper is organized as follows. Section 2 gives problem statement. Designing fuzzy adaptive controllers and nonlinear observers are proposed in Section 3. Section 4 presents simulation results of the proposed controller and finally Section 5 concludes the paper.

2. PRELIMINARIES

Consider the following large scale nonlinear system.

$$\begin{cases} \dot{x}_{i,l} = f_{i,l}(\underline{x}_i) \quad l = 1,2,\dots,n_i - 1 \quad , i = 1,2,\dots,N \\ \dot{x}_{i,n_i} = f_i(\underline{x}_i) + g_i(\underline{x}_i)u_i + m_i(\underline{x}_1, \underline{x}_2, \dots, \underline{x}_N) + d_i(t) \\ y_i = C_i^T \underline{x}_i \end{cases} \tag{1}$$

where $x_{i,l}$ declares l^{th} state of the i^{th} subsystem, n_i is number of the state in i^{th} subsystem, N is number of the subsystems, $\underline{x}_i = [x_{i,1}, \dots, x_{i,n_i}]^T \in R^{n_i}$ is the state vector of the system which is assumed not available for measurement, $u_i \in R$ is the control input, $y_i \in R$ is the system output, $f_{i,l}(\underline{x}_i)$'s and $g_i(\underline{x}_i)$ are unknown smooth nonlinear function, $m_i(\underline{x}_1, \underline{x}_2, \dots, \underline{x}_N)$ is an unknown nonlinear interconnection term, and $d_i(t)$ is bounded disturbance.

The above equation can be revised as

$$\begin{cases} \underline{\dot{x}}_i = A_{i0}\underline{x}_i + \underbrace{(-A_{i0}\underline{x}_i + f_i(\underline{x}))}_{f_i'(\underline{x})} + b_i(f_i(\underline{x}_i) + g_i(\underline{x}_i)u_i + m_i(\underline{x}_1, \underline{x}_2, \dots, \underline{x}_N) + d_i(t)) \\ y_i = C_i^T \underline{x}_i \end{cases} \tag{2}$$

where $f_i(\underline{x}_i)$, A_{i0} and b_i are defined below.

$$\begin{gathered} A_{i0} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n \times n} \\ b_i = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T \in R^n \\ f_i(\underline{x}) = \begin{bmatrix} f_{i,1}(\underline{x}_i) & \cdots & f_{i,n_i-1}(\underline{x}_i) & 0 \end{bmatrix}^T \end{gathered} \tag{3}$$

The control objective is to design an observer based adaptive fuzzy controller for system (1) such that the tracking error and observer error are ultimately bounded while all signals in the closed-loop system remain bounded.

In this paper, we will make the following assumptions concerning the system (1) and the desired trajectory $x_{im}(t)$.

Assumption 1: without loss of generality, it is assumed that the nonzero function $g_i(\underline{x}_i)$ satisfies the following condition:

$$g_i(\underline{x}_i) \geq g_{\min} > 0 \quad \forall(\underline{x}_i, u_i) \in R^{n_i} \times R$$
$$\frac{dg_i(\underline{x}_i)}{dt} \geq g_{dm} \tag{4}$$

$g_{dm} \in R$ is known constant parameter and define later. Furthermore, the following controller and observer design can be reconstructed for $g_{\min} < 0$ in same way.

Assumption 2: The desired trajectory $x_{im}(t)$ is generated by the following desired system.

$$\begin{cases} \dot{\underline{x}}_{im} = A_{i0}\underline{x}_{im} + b_i r_i(t) \\ y_{im} = C_i^T \underline{x}_{im} \end{cases} \tag{5}$$

where $r_i(t)$ is external desired value.

Assumption 3: the interconnection term satisfies the following:

$$\left| m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) \right| \leq \xi_{i0} + \sum_{j=1}^{N} \xi_{ij} \left(\left| C_i^T \underline{\tilde{e}}_i \right| \right) \left\| \underline{\hat{x}}_j \right\| \tag{6}$$

$\xi_{i0} + \sum_{j=1}^{N} \xi_{ij} \left(\left| C_i^T \underline{\tilde{e}}_i \right| \right) \left\| \underline{\hat{x}}_j \right\|$ is unknown upper bound of interaction terms. To use this upper bound in controller design procedure, it uses the $\hat{\zeta}_{ij}$'s as estimation of ζ_{ij} 's that adjusted adaptively.

Assumption 4: the external disturbance satisfies the following property.

$$\left\| d_i(t) \right\|_\infty \leq d_{\max} \tag{7}$$

Consider $\underline{\hat{x}}_i(t)$ as an estimation of $\underline{x}_i(t)$ and the following definitions.

$$\begin{aligned} \underline{e}_i &= \underline{x}_{im} - \underline{x}_i = [e_i \;\; \dot{e}_i \;\; ...e_i^{(n-1)}]^T \\ \underline{\hat{e}}_i &= \underline{x}_{im} - \underline{\hat{x}}_i = [\hat{e}_i \;\; \dot{\hat{e}}_i \;\; ...\hat{e}_i^{(n-1)}]^T \\ \underline{\tilde{e}}_i &= \underline{e}_i - \underline{\hat{e}}_i \end{aligned} \tag{8}$$

where $\underline{e}_i$ stands for the tracking error, $\underline{\hat{e}}_i$ presents the observer error and $\underline{\tilde{e}}_i$ is for the observation error. Consider the following tracking error vector.

$$\underline{e}_i = [e_{i,1}, e_{i,2}, \ldots, e_{i,n_i}]^T \in R^{n_i} \tag{9}$$

Taking the derivative of both sides of the equation (8) we have

$$\begin{cases} \dot{\underline{e}}_i = \dot{\underline{x}}_{im} - \dot{\underline{x}}_i = A_{i0}\underline{x}_{im} + b_i r_i(t) - A_{i0}\underline{x}_i - f_i'(\underline{x}) - b_i\left(f_i(\underline{x}_i) + g_i(\underline{x}_i)u_i + m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) + d_i(t)\right) \\ \underline{e}_{iy} = C_i^T \underline{e}_i \end{cases} \tag{10}$$

Use equation (3) to rewrite the above equation as:

$$\begin{cases} \dot{\underline{e}}_i = A_{i0}\underline{e}_i - f_i'(\underline{x}) + b_i\{r_i(t) - f_i(\underline{x}_i) + g_i(\underline{x}_i)u_i - m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) - d_i(t)\} \\ \underline{e}_{iy} = C_i^T \underline{e}_i \end{cases} \tag{11}$$

where A_{i0} is defined below.

$$\mathrm{A}_{i0}=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\0&0&0&\cdots&0\end{bmatrix}\in R^{n_i\times n_i} \tag{12}$$

To construct the controller, let v_i be defined as

$$\mathrm{v}_i = r_i(t)+\mathrm{k}_{ic}^{\mathrm{T}}\underline{\hat{\mathrm{e}}}_i + v_i' \tag{13}$$

Consider the vector $\mathrm{k}_{ic} = [\mathrm{k}_{i,1}, \mathrm{k}_{i,2}, \ldots, \mathrm{k}_{i,n_i}]^{\mathrm{T}}$ be coefficients of $\psi(s) = s^{n_i} + \mathrm{k}_{i,n_i}\mathrm{s}^{n_i-1}+...+\mathrm{k}_{i,1}$ and chosen so that the roots of this polynomial are located in the open left-half plane. This makes the matrix $\mathrm{A}_{ioc} = \mathrm{A}_{i0} - \mathrm{b}_i\mathrm{k}_{ic}^{\mathrm{T}}$ be Hurwitz.

By adding and subtracting the term $\left(\mathrm{k}_{ic}^{\mathrm{T}}\underline{\hat{\mathrm{e}}}_i + v_i'\right)$ from the right-hand side of equation (14), we obtain

$$\begin{cases}\underline{\dot{\mathrm{e}}}_i = \mathrm{A}_{i0}\underline{\mathrm{e}}_i - \mathrm{b}_i\mathrm{k}_{ic}^{\mathrm{T}}\underline{\hat{\mathrm{e}}}_i - f_i'(\underline{x}) - \mathrm{b}_i\{f_i(\underline{x}_i)+g_i(\underline{x}_i)u_i - v_i + m_i(\underline{x}_1,\underline{x}_2,...,\underline{x}_N)+d_i(t)+v_i'\}\\ \mathrm{e}_{iy} = \mathrm{C}_i^{\mathrm{T}}\underline{\mathrm{e}}_i\end{cases} \tag{14}$$

Invoking the implicit function theorem, it is obvious that the nonlinear algebraic equation $f_i(\underline{x}_i)+g_i(\underline{x}_i)u_i - v_i = 0$ is locally soluble for the input u_i for an arbitrary $(\underline{\mathrm{x}}_i, \mathrm{v}_i)$. Thus, there exists some ideal controller $\mathrm{u}_i^*(\underline{\mathrm{x}}_i, \mathrm{v}_i)$ satisfying the following equality for a given $(\underline{\mathrm{x}}_i, \mathrm{v}_i) \in \mathrm{R}^{n_i}\times \mathrm{R}$:

$$f_i(\underline{x}_i)+g_i(\underline{x}_i)u_i^* - v_i = 0 \tag{15}$$

As a result of the mean value theorem, $f_i(\underline{x}_i)+g_i(\underline{x}_i)u_i$ can be expressed around u_i^* as:

$$f_i(\underline{x}_i)+g_i(\underline{x}_i)u_i = f_i(\underline{x}_i)+g_i(\underline{x}_i)\mathrm{u}_i^* + (\mathrm{u}_i-\mathrm{u}_i^*)g_i(\underline{x}_i) = f_i(\underline{x}_i)+g_i(\underline{x}_i)\mathrm{u}_i^* + \mathrm{e}_{u_i}g_i(\underline{x}_i) \tag{16}$$

Substituting equation (16) into the error equation (14), we get

$$\begin{cases}\underline{\dot{\mathrm{e}}}_i = \mathrm{A}_{i0}\underline{\mathrm{e}}_i - \mathrm{b}_i\mathrm{k}_{ic}^{\mathrm{T}}\underline{\hat{\mathrm{e}}}_i - f_i'(\underline{x}) - \mathrm{b}_i\{e_{iu}g_i(\underline{x}_i)+m_i(\underline{x}_1,\underline{x}_2,...,\underline{x}_N)+d_i(t)+v_i'\}\\ \mathrm{e}_{iy} = \mathrm{C}_i^{\mathrm{T}}\underline{\mathrm{e}}_i\end{cases} \tag{17}$$

3. OBSERVER BASED FUZZY ADAPTIVE CONTROLLER DESIGN

If the function is continuous on compact set, it can be written in compact form as follow:

$$y(x) = w(x)^T\theta \tag{18}$$

where $\theta = \left[y^1 y^2 \ldots y^M\right]$ is a vector of consequent parameters, and $w(x) = [w_1(x)\, w_2(x)\, \ldots w_M(x)]^T$ is a set of fuzzy basis functions.

This section deals with the observer and controller design procedure. To design observer for the mentioned system in equation (17), this paper proposes the following observer error.

$$\begin{cases} \dot{\underline{\hat{e}}}_i = \underbrace{(\mathrm{A}_{i0}\text{-}\mathrm{b}_i \mathrm{k}_{ic}^{\mathrm{T}})}_{\mathrm{A}_{ioc}} \underline{\hat{e}}_i + \mathrm{K}_{i0} C_i^T \underline{\tilde{e}} + b_i k_{ino} (\underline{\tilde{e}}_i, \underline{\hat{e}}_i) \left| C_i^T \underline{\tilde{e}}_i \right| \\ \hat{e}_{iy} = \mathrm{C}_i^{\mathrm{T}} \underline{\hat{e}}_i \end{cases} \tag{19}$$

where K_{io}, k_{ino} are the linear observer gain and the nonlinear observer gain respectively. K_o is selected to make sure that the characteristic polynomial of $(\mathrm{A}_{ioo} = \mathrm{A}_{io}\text{-}\mathrm{K}_{i0} C_i^T)$ is Hurwitz. Defining the observation error $\underline{\tilde{e}}_i = \underline{e}_i - \underline{\hat{e}}_i$. Subtracting (17) from (19) yields

$$\begin{cases} \dot{\underline{\tilde{e}}}_i = \underbrace{(\mathrm{A}_{i0}\text{-}\mathrm{K}_{i0} C_i^T)}_{A_{oo}} \underline{\tilde{e}}_i - f_i'(\underline{x}) - \mathrm{b}_i \{ e_{iu} g_i(\underline{x}_i) + d_i(t) + v_i' + m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) + k_{ino}(\underline{\tilde{e}}_i, \underline{\hat{e}}_i) \left| C_i^T \underline{\tilde{e}}_i \right| \} \\ \tilde{e}_{iy} = \mathrm{C}_i^{\mathrm{T}} \underline{\tilde{e}}_i \end{cases} \tag{20}$$

The output error dynamics of the above equation can be given as:

$$\tilde{e}_{iy} = H_i(s)\{ f_i'(\underline{x}) + \mathrm{b}_i \{ e_{iu} g_i(\underline{x}_i) + m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) + d_i(t) + v_i' + k_{ino}(\underline{\tilde{e}}_i, \underline{\hat{e}}_i) \left| C_i^T \underline{\tilde{e}}_i \right| \} \} \tag{21}$$

Where

$$\mathrm{H}_i(\mathrm{s}) = -\,\mathrm{C}_i^{\mathrm{T}} \left(\mathrm{sI}\text{-}(\mathrm{A}_{i0}\text{-}\mathrm{K}_{i0} C_i^T) \right)^{-1} B_i \tag{22}$$

Where B_i is the identity matrix, $\mathrm{H_i(s)}$ is a known stable transfer function. In order to use the SPR-Lyapunov design approach, equation (21) is rewritten as

$$\tilde{e}_{iy} = H_i(s) L_i(s) \{ f_{if}'(\underline{x}) + \mathrm{b}_i \{ e_{iu} g_i(\underline{x}_i) + d_{if}(t) + v_{if}' + m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) + k_{inof}(\underline{\tilde{e}}_i, \underline{\hat{e}}_i) \left| C_i^T \underline{\tilde{e}}_i \right| \} \} \tag{23}$$

where $f_{if}'(\underline{x}_i) = f_i'(\underline{x}_i)$, $f_{iu_\lambda f} = L_i(s)^{-1} f_{iu_\lambda}$, $k_{inof} = L_i(s)^{-1} k_{ino}$, $v_{if}' = L_i(s)^{-1} v_i'$, $d_{if}(t) = L_i(s)^{-1} d_i(t)$. $\mathrm{L_i(s)}$ is chosen so that $L_i(s)^{-1}$ is a proper stable transfer function and $H_i(s) L_i(s)$ is a proper strictly-positive-real (SPR) transfer function. Let $\mathrm{L_i(s) = s^m + b_{i1} s^{m-1} + \cdots + b_{im}}$ where $(\mathrm{m = n - 1})$.

The state-space realization of (23) can be written as

$$\begin{cases} \dot{\underline{\tilde{e}}}_{is} = A_{ioo} \underline{\tilde{e}}_{is} - B_{is} f_{if}'(\underline{x}) - \mathrm{b}_{is} \{ e_{iu} g_i(\underline{x}_i) + d_{if}(t) + m_{if}(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) + v_{if}' + k_{inof}(\underline{\tilde{e}}_i, \underline{\hat{e}}_i) \left| \mathrm{C}_i^{\mathrm{T}} \underline{\tilde{e}}_{is} \right| \} \\ \tilde{e}_{iy} = \mathrm{C}_{is}^{\mathrm{T}} \tilde{e}_{is} \end{cases} \tag{24}$$

The ideal controller can be represented as:

$$u_i^* = f_i(\underline{z}_i) + \varepsilon_{iu} \tag{25}$$

where $z_i = [x_i, v_i]^T$ and $f_i(\underline{z}) = \theta_{i1}^* w_{i1}(\underline{z})$, and θ_{i1}^* and $w_{i1}(\underline{z}_i)$ are consequent parameters and a set of fuzzy basis functions, respectively. ε_{iu} is an approximation error that satisfies $\left| \varepsilon_{iu} \right| \le \varepsilon_{\max}$ and $\varepsilon_{\max} > 0$. Denote the estimate of θ_{i1}^* as θ_{i1} and u_{irob} as a robust controller to compensate approximation error, uncertainties, disturbance and interconnection term. To rewrite the controller given in (25) as:

$$u_i = \theta_{i1}^T w_{i1}(\underline{z}_i) + u_{irob} + \underline{\hat{e}}_i^T P_{i2} \mathrm{K}_{i0} \tag{26}$$

Consider $\xi_{ij}\left(\left|C_i^T \underline{\tilde{e}}_i\right|\right)=\eta_{ji}\left\|C_i^T \underline{\tilde{e}}_i\right\|$ and then u_{irob} is defined below.

$$u_{irob}=sign(C_{is}^T\underline{\tilde{e}}_{is})(\frac{N}{2g_{\min}}\left|C_{is}^T\underline{\tilde{e}}_{is}\right|+\frac{\hat{\xi}_{i0}}{g_{\min}}+\frac{1}{2g_{\min}}\sum_{j=1}^{N}\hat{\eta}_{ij}\left\|\underline{\hat{x}}_i\right\|^2\left|C_{is}^T\underline{\tilde{e}}_{is}\right|+u_{icom}+\frac{u_{ir}}{g_{\min}} \\ +\frac{\hat{v}_i'}{g_{\min}}+\hat{k}_{inof}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)(\frac{\left|C_{is}^T\underline{\tilde{e}}_{is}\right|}{g_{\min}}+\left|\underline{\hat{e}}_i^T P_{i1}b_i\right|)) \tag{27}$$

In the above, $\theta_{i1}^T w_{i1}(\underline{z})$ approximates the ideal controller, $\hat{\xi}_{i0}+\frac{1}{2}\sum_{j=1}^{N}\hat{\eta}_{ij}\left\|\underline{\hat{x}}_j\right\|^2\left|C_{is}^T\underline{\tilde{e}}_{is}\right|$ tries to estimate the interconnection term, u_{icom} compensates for approximation errors and uncertainties, u_{ir} is designed to compensate for bounded external disturbances, $\hat{k}_{inof}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)(\left|C_{is}^T\underline{\tilde{e}}_{is}\right|/g_{\min}+\left|\underline{\hat{e}}_i^T P_{i1}b_i\right|))$ tries to estimate the nonlinear gain of the observer, and $\hat{v}_i'$ is estimation of v_i'. Define error vector $\tilde{\theta}_{i1}=\theta_{i1}-\theta_{i1}^*$ and use (26) and (27) to rewrite the error equation (17) as:

$$\begin{cases}\dot{\underline{e}}_i=A_{i0}\underline{e}_i-b_i k_{ic}^T\underline{\hat{e}}_i-f_i'(\underline{x})-b_i\{(\tilde{\theta}_{i1}^T w_{i1}(\underline{z}_i)+u_{irob}-\varepsilon_{iu})g_i(\underline{x}_i)+m_i(\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_N)+d_i(t)+v_i'\} \\ e_{iy}=C_i^T\underline{e}_i\end{cases} \tag{28}$$

Based on equation (26) and (27), the state-space realization of the equation (24) can be written as

$$\begin{cases}\dot{\underline{\tilde{e}}}_{is}=A_{ioo}\underline{\tilde{e}}_{is}-B_{is}f_{if}'(\underline{x})-b_{is}\{(\tilde{\theta}_{i1}^T w_{i1}(\underline{z}_i)+u_{irob}-\varepsilon_{iu})g_i(\underline{x}_i)+d_{if}(t)+v_{if}'+k_{inof}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_i^T\underline{\tilde{e}}_i\right|\} \\ \tilde{e}_{iy}=C_{is}^T\underline{\tilde{e}}_{is}\end{cases} \tag{29}$$

Consider the following update laws.

$$\begin{aligned}
\dot{\hat{k}}_{ino} &= \gamma_{iko}(\frac{\left|C_{is}^T\underline{\tilde{e}}_{is}\right|^2}{g_{\min}}+\left|C_{is}^T\underline{\tilde{e}}_{is}\right|\left|b_i^T P_{i2}\underline{\hat{e}}_i\right|) \\
\dot{\theta}_{i1} &= \Gamma_1 C_{is}^T\underline{\tilde{e}}_{is}w_{i1}(\underline{z}_i) \\
\dot{\hat{\xi}}_{i0} &= \frac{\gamma_{\xi_{i0}}}{g_{\min}}\left|C_{is}^T\underline{\tilde{e}}_{is}\right| \\
\dot{\hat{\eta}}_{ji} &= \frac{\gamma_{\eta_{ji}}}{2g_{\min}}\left|C_{is}^T\underline{\tilde{e}}_{is}\right|\left\|\underline{\hat{x}}_i\right\|^2 \\
\dot{u}_{ir} &= \gamma_{u_{ir}}\left|C_{is}^T\underline{\tilde{e}}_{is}\right| \\
\dot{u}_{icom} &= \gamma_{u_{icom}}\left|C_{is}^T\underline{\tilde{e}}_{is}\right| \\
\dot{\hat{v}}_i' &= \gamma_{\hat{v}_i'}\left|C_{is}^T\underline{\tilde{e}}_{is}\right|
\end{aligned} \tag{30}$$

where $\Gamma_1=\Gamma_1^T>0,\gamma_{u_{ir}},\gamma_{\eta_{ji}},\gamma_{u_{icom}},\gamma_{\hat{v}_i'},\gamma_{iko}>0$ are constant parameters.

Theorem 2: consider the error dynamical system given in (19) and (29) for the large scale system (1) satisfying assumption (1), interconnection term satisfying assumption (3), the external disturbances satisfying assumption (4), and a desired trajectory satisfying assumption (2), then the controller structure given in (26), (27) with adaptation laws (30) makes the tracking error and the observer error converge asymptotically to the origin and all signals in the closed loop system be bounded.

Proof: consider the following Lyapunov function.

$$V=\sum_{i=1}^{N}\frac{1}{2}\Big(\frac{1}{g_{iu_{\lambda f}}}\underline{\tilde{e}}_{is}^{T}P_{i1}\underline{\tilde{e}}_{is}+\underline{\hat{e}}_{i}^{T}P_{i2}\underline{\hat{e}}_{i}+\tilde{\theta}_{i1}^{T}\Gamma_{1}^{-1}\tilde{\theta}_{i1}+\frac{\tilde{\xi}_{i0}^{2}}{\gamma_{\xi_{i0}}}+\frac{\sum_{j=1}^{N}\tilde{\eta}_{ji}^{2}}{\gamma_{\eta_{ji}}}+\frac{\tilde{u}_{ir}^{2}}{\gamma_{u_{ir}}}+\frac{\tilde{u}_{icom}^{2}}{\gamma_{u_{icom}}}+\frac{\tilde{k}_{ino}^{2}}{\gamma_{iko}}+\frac{\tilde{v}_{i}'^{2}}{\gamma_{\tilde{v}_{i}'}}\Big) \tag{31}$$

where $\tilde{\theta}_{i1}=\theta_{i1}-\theta_{i1}^{*}$, $\tilde{u}_{ir}=u_{ir}-d_{\max}$, $\tilde{u}_{icom}=u_{icom}-\varepsilon_{\max}$, $\tilde{k}_{ino}=\hat{k}_{ino}-k_{ino}$, $\tilde{\eta}_{ji}=\hat{\eta}_{ji}-\eta_{ji}$, $\tilde{\xi}_{i0}=\hat{\xi}_{i0}-\xi_{i0}$ and $\tilde{v}_{i}'=\hat{v}_{i}'-|v_{i}'|$. After some manipulation on the time derivative of the Lyapunov function and based on equations (19), (26), (27), (29) and (30).

$$\dot{V}\leq\sum_{i=1}^{N}-\frac{1}{2f_{iu_{\lambda f}}}\underline{\tilde{e}}_{is}^{T}\underbrace{\left(Q_{i1}+\frac{\dot{f}_{iu}}{f_{iu}}P_{i1}\right)}_{M_{i}}\underline{\tilde{e}}_{is}-\frac{1}{2}\underline{\hat{e}}_{i}^{T}Q_{i2}\underline{\hat{e}}_{i}+\frac{1}{f_{\min}}\left\|f_{i}'^{T}(\underline{x})\right\|\left\|B_{is}^{T}P_{i1}\right\|\|\underline{\tilde{e}}_{is}\| \tag{32}$$

Based on boundedness of the reference signals and $\left\|f_{i}'^{T}(\underline{x})\right\|\leq c_{1}\|x_{i}\|+c_{2}$, the following inequality has been satisfied.

$$\begin{aligned}\left\|f_{i}'^{T}(\underline{x})\right\|&\leq c_{1}\|\underline{x}_{i}\|+c_{2}\\&\leq c_{1}\Big\|\underbrace{\underline{x}_{i}-\underline{\hat{x}}_{i}}_{\underline{\tilde{e}}_{is}}+\underbrace{\underline{\hat{x}}_{i}-\underline{x}_{im}}_{\underline{\hat{e}}_{i}}+\underline{x}_{im}\Big\|+c_{2}\\&\leq c_{1}\|\underline{\tilde{e}}_{is}\|+c_{1}\|\underline{\hat{e}}_{i}\|+\underbrace{c_{1}\|\underline{x}_{im}\|+c_{2}}_{\leq c_{2}'}\end{aligned} \tag{33}$$

Using (33), the equation (32) can be written as:

$$\begin{aligned}\dot{V}\leq&-\frac{1}{f_{\min}}\lambda_{\min}(M_{i})\|\underline{\tilde{e}}_{is}\|^{2}-\frac{1}{2}\lambda_{\min}(Q_{i2})\|\underline{\hat{e}}_{i}\|^{2}+\frac{1}{f_{\min}}c_{1}\|\underline{\tilde{e}}_{is}\|^{2}\left\|B_{is}^{T}P_{i1}\right\|\\&+\frac{1}{f_{\min}}c_{1}\|\underline{\hat{e}}_{i}\|\|\underline{\tilde{e}}_{is}\|\left\|B_{is}^{T}P_{i1}\right\|+\frac{1}{f_{\min}}c_{2}'\|\underline{\tilde{e}}_{is}\|\left\|B_{is}^{T}P_{i1}\right\|\end{aligned} \tag{34}$$

The following inequality can be written as:

$$\begin{aligned}\dot{V}\leq&-\frac{\|\underline{\tilde{e}}_{is}\|}{f_{\min}}\left(\lambda_{\min}(M_{i})\|\underline{\tilde{e}}_{is}\|-c_{1}\|\underline{\tilde{e}}_{is}\|\left\|B_{is}^{T}P_{i1}\right\|-c_{2}'\left\|B_{is}^{T}P_{i1}\right\|\right)\\&-\left(\frac{1}{2}\lambda_{\min}(Q_{i2})\|\underline{\hat{e}}_{i}\|-\frac{1}{f_{\min}}c_{1}\|\underline{\tilde{e}}_{is}\|\left\|B_{is}^{T}P_{i1}\right\|\right)\|\underline{\hat{e}}_{i}\|\end{aligned} \tag{35}$$

Choosing appropriate M_{i}, Q_{i2}, we can guarantee that $\dot{V}$ is negative as long as $\underline{\tilde{e}}_{is},\underline{\hat{e}}_{i}$ is outside the compact set Ω_{e} defined as

$$\Omega_{e}=\left\{\underline{\tilde{e}}_{is},\underline{\hat{e}}_{i}\;\middle|\;\begin{aligned}&\|\underline{\tilde{e}}_{is}\|\leq\frac{c_{2}'\left\|B_{is}^{T}P_{i1}\right\|}{\lambda_{\min}(M_{i})-c_{1}\left\|B_{is}^{T}P_{i1}\right\|}\\&\|\underline{\hat{e}}_{i}\|\leq\frac{2c_{1}\left\|B_{is}^{T}P_{i1}\right\|c_{2}'\left\|B_{is}^{T}P_{i1}\right\|}{\lambda_{\min}(Q_{i2})f_{\min}\left(\lambda_{\min}(M_{i})-c_{1}\left\|B_{is}^{T}P_{i1}\right\|\right)}\end{aligned}\right\} \tag{36}$$

According to standard Lyapunov theorem, we conclude that observation error and accordingly the observer error and the tracking error is ultimately bounded and $\underline{\tilde{e}}_{is}, \underline{\hat{e}}_i$ will converge to Ω_e. In addition, the boundedness of and the coefficient parameters is guaranteed. It completes the proof.
Q.E.D.

4. SIMULATION RESULTS

In this section, we apply the proposed observer based decentralized fuzzy model reference adaptive controller to the following large scale system.

$$\begin{cases} \dot{x}_{11} = \sin(x_{11}) + x_{12} - x_{11} \\ \dot{x}_{12} = \sin(x_{11}) + 1 - x_{12} + 200u_1 + 4\sin(x_{21}) + d_1(t) \\ y_1 = x_{11} \end{cases}$$
$$\begin{cases} \dot{x}_{21} = \sin(x_{21}) + x_{22} - x_{21} \\ \dot{x}_{22} = \sin(x_{21}) + 1 - x_{22} + 200u_2 + 4\sin(x_{11}) + d_2(t) \\ y_2 = x_{21} \end{cases} \tag{37}$$

It has been considered that the desired value of the outputs are $r_1 = 2\sin(\pi t) + 2\sin(3\pi t)$ and $r_2 = 2\sin(1.8\pi t) + 2\sin(3.6\pi t)$. Furthermore, it is assumed that $d_1(t) = \sin(200\pi t)$ and $d_2(t) = \sin(120\pi t)$.

Now we applied the proposed controller defined in (35), (36) to the system that mentioned in equation (1). Based on the experts' knowledge, Let define $x_i = [x_{i1}, x_{i2}]^T$, $z_i = [x_{i1}, x_{i2}, v_i]^T$ and the states of the subsystems are in the range of $[5, -5]$, furthermore v_i are defined over $[-45, 45]$. For each fuzzy system input, we define 6 membership functions over the defined sets. Consider that all of the membership functions are defined by the Gaussian function $\mu_j(\chi) = \exp((\chi - c)^2 / 2\delta^2)$, where c is center of the membership function and δ is its variance. We assume that the initial value of controller parameters be zero. Furthermore, it has been assumed that $f_{\min} = 1$, $\Gamma_1 = 10$, $\gamma_{\xi_{i0}} = 2$, $\gamma_{\xi_{ij}} = 2$, $\gamma_{u_{icom}} = 2$, $\gamma_{u_{ir}} = 5$, $\gamma_{\hat{v}_i'} = 2$ and $\gamma_{iko} = 10$.

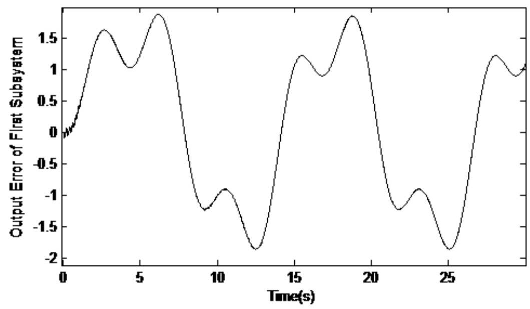

Figure 1. Performance of the proposed controller in first subsystem

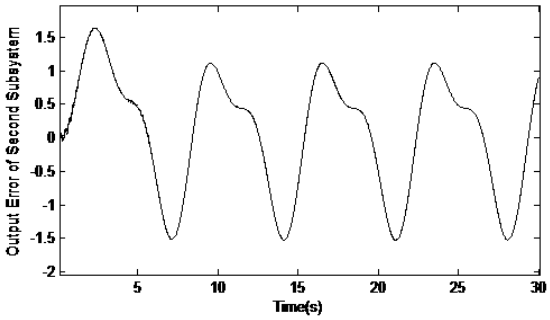

Figure 2. Performance of the proposed controller in second subsystem

As shown in figures (1) and (2), it is obvious that the performance of the proposed controller is promising. Figure (3) and (4) show the estimation of the first and second states of the first subsystem with their desired value.

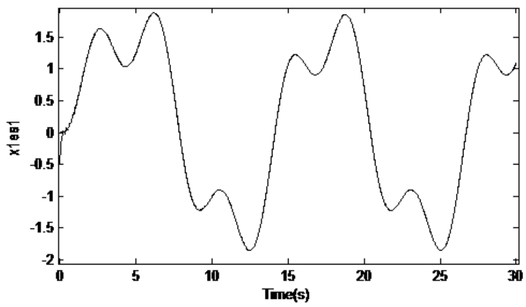

Figure 3. The estimation of the first state of the first subsystem and the desired value

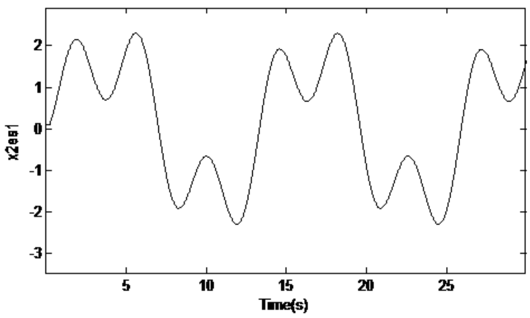

Figure 4. The estimation of the second state of the first subsystem

The performance of the proposed observer on the second subsystem and their desired trajectories are shown in figure (5) and (6).

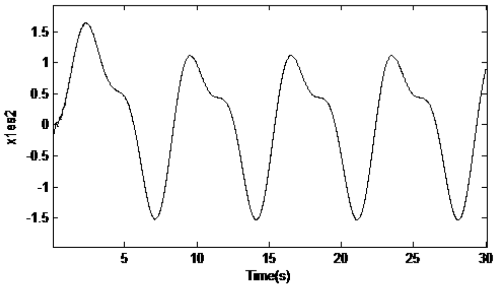

Figure 5. The estimation of the first state of the second subsystem and the desired value

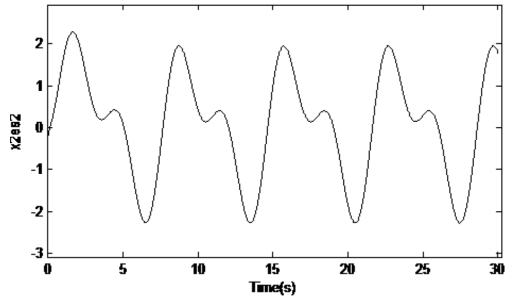

Figure 6. the estimation of the second state of the second subsystem

As shown in figures (3) through (6), it is obvious that the nonlinear state observer can generate the estimated states and perform exactly. Moreover, it is also clear that the output of the system converge to the desired value. The stability of the closed loop, the boundedness of the tracking error and the observer error, and robustness against external disturbance and approximation error are the merits of the proposed controller and observer.

5. CONCLUSION

The proposed observer and controller are applied on large scale non-canonical nonlinear system. The advantages of the proposed method are as the boundedness of the tracking and observer error, robustness against external disturbances and approximation errors.

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