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A fuzzy observer synthesis to state and fault estimation for Takagi-Sugeno implicit systems

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ABSTRACT

The present paper inquire a fuzzy observer design issue for continuous time Takagi-Sugeno aiming at estimating both of fault and state in case of unmeasurable premise variables. Thanks to singular value decomposition (SVD) approach, the fuzzy observer is synthesized in explicit form. The developed method is based on augmented structure that assemble state and actuator fault, sensor fault as well as their sequential derivatives is enjoined to institute the destined observer. The Lyapunov function is analyzed to assume the exponential stability of the studied observer, furthermore convergence conditions are expressed as linear matrix inequalities (LMIs) form. The applicability of the nominated theoritical result is elucidated and confiremed through numerical simulation using an example of an implicit fuzzy model.

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1. INTRODUCTION

Many chemical and physical processes are governed by both differential and algebraic equations (DAEs) see for instance [1]–[3] and the references therein. These systems called implicit systems, singular systems or descriptor systems exemplify physical constraint that cannot be qualified by usual models. Otherwise, as it recognized that the explicit Takagi-Sugeno (T-S) fuzzy model [4], [5] has received much attention. Due to the fact that a various industrial processes are complex and strongly nonlinear, T-S approach remains an accurate and robust gadget to control and model these laborious and nonlinear systems see [6]. In fact, to facilitate the design of controller as well as observer for arduous nonlinear systems, numerous representations exist of T-S models which gather the linear local models and a nonlinear activation function. Among the benefits of translating the nonlinear system into a T-S model is that the last permit exploiting the direct second Lyapunov method in addition stability conditions are earned as linear matrix inequalities (LMIs) whom can be computed by means of convex optimization technique. Another advantage is the ability to expanded many technique developed by switching from nonlinear systems to linear ones, see e.g. [7], [8]. Control likewise observation of T-S models have been the aim of several studies [9]–[11]. The T-S explicit model [4] represent a partical status of the T-S implicit model. In fact, the T-S fuzzy model [4] can be prolonged as a fuzzy implicit model [12], [13]. The application of T-S fuzzy state observation for dynamics implicit models has been the purpose of various studies. A Luenberger observer for T-S implicit system using a recursive least square method is proposed by Ilhem *et al.* [14]. A novel approach dedicated to design observer for continuous-time T-S implicit model when premise variables are unmeasurable has been developed in [15]. The latter approach is inspired by the separation of dynamic and static relations. Another strategy based on singular value decomposition method is described in [16], it investigates an observer design when input is unknown for a T-S fuzzy implicit system permitting to jointly estimate both of unknown states and unknown inputs.

In this perspective, the aim here is to consider the issue of fuzzy observer design for simultaneous state and fault estimation (SSFE) for a class of continuous-time T-S implicit systems (CTSISs). Notice that, fault detection problems represent actually a necessity to preserve the systems and the human potential and have attracted considerable attention of paramount research areas in these last decades. Due to the fact that the modern technological systems are easily prone by faults which change the behaviour of the systems, and induce a big loss in terms of production, staff and equipment safety, the use of a diagnostic system is highly appreciated and promoted, moreover as cited in [17], researchers have paid much effort to avoid any destruction can affect processes by detecting and estimating of faults. Notice that this domain of application has been widely tackled in numerous publications. Moreover, the application of fuzzy observer design such like diagnosis as well as detection are discussed in [18]–[28]. The main contribution of this paper is to provide a news solution to state and fault estimation problem with unmeasurable premise variable based on singular value decomposition (SVD) approach for a class of T-S implicit models the fuzzy observer are designed. To accomplish the desired observer mission the exponential stability of the system are studied through Lyapunov function, likewise the convergence conditions are resolved via LMIs formulations with integration of $\mathcal{L}_2$-gain technique.

The remainder of this note is composed of: Section 2 expounds the mathematical representation of CTSISs in existence of sensor fault and actuator fault. Section 3 discuss the major result obtained on the novel fuzzy observer aimed at estimating jointly both of actuator fault, sensor fault and state. Section 4 illustates the fulfillment of the enhanced result in numerical simulations via a fuzzy applicable model. Section 5 concludes the paper. In the sequel and throughout the paper, we adopt the following notations:

− $X < 0$ (or $X > 0$) expounds that X is a matrix of symmetry and negative definite or positive definite .
− X^{-1} and X^T is respectively a representation of the inverse and transpose of X.
− I (or 0) obvious the matrix of identity (or zero matrix) with appropriate dimension.
− $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ denote respectively the spaces of n-dimensional real vectors and $n \times m$ real matrices.
− The symbol $*$ designs the symmetric terms.
− $\begin{pmatrix} X & * \\ Z & Y \end{pmatrix} = \begin{pmatrix} X & Z^T \\ Z & Y \end{pmatrix}$ and $\sum_{i,j=1}^{r} \lambda_i(\eta)\lambda_j(\eta) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\eta)\lambda_j(\eta)$

2. PROBLEM FORMULATION

The mathematical representation of CTSIS affected by actuator fault and sensor fault in case of unmeasurable premise variable is considered :

$$\begin{cases} \Gamma\dot{\xi} = \sum_{j=1}^{r} \lambda_j(\eta)(A_j\xi + B_j u + \Lambda_{aj}\gamma_a) \\ y = C\xi + Du + \Omega_a\gamma_a + \Lambda_s\gamma_s \end{cases} \tag{1}$$

where $\xi \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^m$ is the input vector, $y \in \mathbf{R}^p$ is the output vector. $\gamma_a \in \mathbf{R}^{n_a}$ is the actuator fault and $\gamma_s \in \mathbf{R}^{n_s}$ is the sensor fault. $A_j \in \mathbf{R}^{n\times n}$, $B_j \in \mathbf{R}^{n\times m}$, $C \in \mathbf{R}^{p\times n}$, $D \in \mathbf{R}^{p\times m}$, $\Omega_a \in \mathbf{R}^{p\times n_a}$, $\Gamma \in \mathbf{R}^{n\times n}$ such that $rank(\Gamma) < n$, $\Lambda_{aj} \in \mathbf{R}^{n\times n_a}$, $\Lambda_s \in \mathbf{R}^{p\times n_s}$, are real known constant matrices. η is the premise variables vector. r is the number of the sub-systems. The $\lambda_j(\eta)$, (j=1,....,r) are the activation function that gather the contribution of all sub-systems.

$$\begin{cases} \Gamma\dot{\xi} = A_j\xi + B_j u + \Lambda_{aj}\gamma_a \\ y = C\xi + Du + \Omega_a\gamma_a + \Lambda_s\gamma_s \end{cases} \tag{2}$$

and satisfy the following convex sum properties:

$$0 \leq \lambda_j(\eta) \leq 1 \;\;\&\;\; \sum_{j=1}^{r} \lambda_j(\eta) = 1 \tag{3}$$

Let us define the vector given by:

$$\gamma^T \;\; = [\gamma_a^T \quad \gamma_s^T] \tag{4}$$

In this way, the system (1) can be rewritten in the suitable structure:

$$\begin{cases} \Gamma\dot{\xi} & = & \sum_{j=1}^{r} \lambda_j(\eta)(A_j\xi + B_ju + \Phi_j\gamma) \\ y & = & C\xi + Du + \Psi\gamma \end{cases} \tag{5}$$

where;

$$\Phi_j = [\Lambda_{aj} \quad 0] \;\;\&\;\; \Psi = [\Omega_a \quad \Lambda_s] \tag{6}$$

Assumption 1: The studied fault γ is assumed of the proppposed structure :

$$\gamma \quad = \quad \sigma_0 + \sigma_1 t + \ldots + \sigma_\alpha t^\alpha \tag{7}$$

where σ_j, $j = 0,\ 1,\ \ldots,\ \alpha$ are a constant terms supposed to be unknown
Let:

$$\mu_j \quad = \quad \gamma^{(j-1)}; \quad j = 1, \ldots, \alpha + 1 \tag{8}$$

Then

$$\begin{cases} \dot{\mu}_j & = & \mu_{j+1}; \quad j = 1, \ldots, \alpha \\ \dot{\mu}_{\alpha+1} & = & 0 \end{cases} \tag{9}$$

In the objectif of jointly estimating both of state, actuator fault and sensor fault, an auxialary augmented system using (5) and (9) is constructed:

$$\begin{cases} E\dot{x} & = & \sum_{j=1}^{r} \lambda_j(\eta)(\tilde{A}_jx + \tilde{B}_ju) \\ y & = & \tilde{C}x + Du \end{cases} \tag{10}$$

where;

$$\begin{cases} x^T & = & [\xi^T \;\; \mu_1^T \;\; \cdots \;\; \mu_\alpha^T] \;\; ; \;\; \tilde{C} = [C \;\; \Psi \;\; 0 \;\; \ldots \;\; 0] \;\; ; \;\; \tilde{B}_j^T = [B_j^T \;\; 0 \;\; \cdots \;\; 0] \\ \tilde{A}_j & = & \begin{pmatrix} A_j & \Phi_j & 0 & \cdots & 0 \\ 0 & 0 & I & & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & & 0 & 0 & I \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \;\; ; \quad E = \begin{pmatrix} \Gamma & 0 \\ 0 & I \end{pmatrix} \end{cases} \tag{11}$$

Before giving the main result, the following Hypotheses are assumed:

– H_1) $(E, \tilde{A}_j)$ is regular, i.e. $det(sE - \tilde{A}_j) \neq 0 \; \forall s \in \mathbf{C}$

– H_2) All sub-models (10) are impulse observable and detectable.

– H_3) $rank(\begin{bmatrix} E \\ \tilde{C} \end{bmatrix}) = n_1 = n + (\alpha+1)(n_a + n_s)$

In the basis of the formulation H_3), a constant matrices $\Omega_1 \in \mathbb{R}^{n_1 \times n_1}$, $\Omega_2 \in \mathbb{R}^{n_1 \times p}$,$\Omega_3 \in \mathbb{R}^{p \times n_1}$,$\Omega_4 \in \mathbb{R}^{p \times p}$ are establish via the singular value decomposition of $[E^T \;\; \tilde{C}^T]^T$:

$$\left\{ \begin{array}{rcl} \Omega_1 E + \Omega_2 \tilde{C} & = & I \\ \Omega_3 E + \Omega_4 \tilde{C} & = & 0 \end{array} \right. \tag{12}$$

3. MAIN RESULT

Based on the SVD approach and using the transformation of CTSIS (1) into the form (10), the propounded fuzzy observer is taken as:

$$\left\{ \begin{array}{rcl} \dot{z} & = & \sum_{j=1}^{r} \lambda_j(\hat{\eta})(M_j z + J_{1j} y + J_{2j} y + H_j u) \\ \hat{x} & = & z + \Omega_2 y + R\Omega_4 y \end{array} \right. \tag{13}$$

where $\hat{x}$ and $\hat{\eta}$ denote respectively the estimate of augmented state vector x and the premise variable η.
M_j, J_{1j}, J_{2j}, H_j and R a considered matrices to be unknown with calculated dimensions, obtained in the aim of a close exponential convergence of $\hat{x}$ to x.
Let us define the state estimation error:

$$\varepsilon \;=\; x - \hat{x} \tag{14}$$

From (10), (12) and (13), (14) becomes:

$$\varepsilon \;=\; (\Omega_1 + R\Omega_3)Ex - z \tag{15}$$

Then, by using (10) and (13), The dynamical error is governed by:

$$\dot{\varepsilon} \;=\; \sum_{j=1}^{r} \lambda_j(\eta)(\Omega_1 + R\Omega_3)(\tilde{A}_j x + \tilde{B}_j u) - \sum_{j=1}^{r} \lambda_j(\hat{\eta})(M_j z + J_{1j} y + J_{2j} y + H_j u) \tag{16}$$

By applying (10), (15), (16) is modified:

$$\dot{\varepsilon} \;=\; \sum_{j=1}^{r} \lambda_j(\eta)(\Omega_1 + R\Omega_3)(\tilde{A}_j x + \tilde{B}_j u) + \sum_{j=1}^{r} \lambda_j(\hat{\eta})(M_j \varepsilon - N_j x - \tilde{H}_j u) \tag{17}$$

where

$$\left\{ \begin{array}{rcl} N_j & = & M_j(\Omega_1 + R\Omega_3)E + J_{1j}\tilde{C} + J_{2j}\tilde{C} \\ \tilde{H}_j & = & H_j + J_{1j} D + J_{2j} D \end{array} \right. \tag{18}$$

Provided the matrices M_j, J_{1j}, J_{2j}, $\tilde{H}_j$ and R satisfy:

$$N_j \;=\; (\Omega_1 + R\Omega_3)\tilde{A}_j \tag{19}$$

$$\tilde{H}_j \;=\; (\Omega_1 + R\Omega_3)\tilde{B}_j \tag{20}$$

Then (17) is well defined and taking the new relation:

$$\dot{\varepsilon} = \sum_{j=1}^{r}\lambda_j(\hat{\eta})M_j\varepsilon + \sum_{j=1}^{r}(\lambda_j(\eta) - \lambda_j(\hat{\eta}))(\Omega_1 + R\Omega_3)(\tilde{A}_j x + \tilde{B}_j u) \tag{21}$$

Taking into account (12), (18), (19) and (20), we infer:

$$M_j = (\Omega_1 + R\Omega_3)\tilde{A}_j - J_{2j}\tilde{C} + (M_j(\Omega_2 + R\Omega_4) - J_{1j})\tilde{C} \tag{22}$$

Taking :

$$J_{1j} = M_j(\Omega_2 + R\Omega_4) \tag{23}$$

The constraint (22) leads to:

$$M_j = (\Omega_1 + R\Omega_3)\tilde{A}_j - J_{2j}\tilde{C} \tag{24}$$

Now, using the fact that:

$$\begin{cases} \sum_{i=1}^{r}(\lambda_i(\eta) - \lambda_i(\hat{\eta}))\tilde{A}_i = \sum_{i,j=1}^{r}\lambda_i(\eta)\lambda_j(\hat{\eta})\Delta\tilde{A}_{ij} \\ \sum_{i=1}^{r}(\lambda_i(\eta) - \lambda_i(\hat{\eta}))\tilde{B}_i = \sum_{i,j=1}^{r}\lambda_i(\eta)\lambda_j(\hat{\eta})\Delta\tilde{B}_{ij} \end{cases} \tag{25}$$

Where $\Delta\tilde{A}_{ij} = \tilde{A}_i - \tilde{A}_j$ and $\Delta\tilde{B}_{ij} = \tilde{B}_i - \tilde{B}_j$

Based on (25) and $\sum_{i=1}^{r}\lambda_i(\eta) = 1$, (21) is simplified as:

$$\dot{\varepsilon} = \sum_{i,j=1}^{r}\lambda_i(\eta)\lambda_j(\hat{\eta})(M_j\varepsilon + \Upsilon_{ij}x + \Psi_{ij}u) \tag{26}$$

with

$$\begin{cases} \Upsilon_{ij} = (\Omega_1 + R\Omega_3)\Delta\tilde{A}_{ij} \\ \Psi_{ij} = (\Omega_1 + R\Omega_3)\Delta\tilde{B}_{ij} \end{cases} \tag{27}$$

Introducing the augmented state vector:

$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon \\ x \end{pmatrix} \tag{28}$$

From (10) and (26), we have:

$$\begin{cases} \bar{E}\dot{\bar{\varepsilon}} = \sum_{i,j=1}^{r}\lambda_i(\eta)\lambda_j(\hat{\eta})(\Pi_{ij}\bar{\varepsilon} + \Lambda_{ij}u) \\ \varepsilon = S\bar{\varepsilon} \end{cases} \tag{29}$$

where

$$\begin{cases} \bar{E} = \begin{pmatrix} I & 0 \\ 0 & E \end{pmatrix} \; ; \; S = (I \;\; 0) \\ \Pi_{ij} = \begin{pmatrix} M_j & \Upsilon_{ij} \\ 0 & \tilde{A}_i \end{pmatrix} \; ; \; \Lambda_{ij} = \begin{pmatrix} \Psi_{ij} \\ \tilde{B}_i \end{pmatrix} \end{cases} \tag{30}$$

Theorem 1: Under assumption 1, and hypotheses H_1), H_2), H_3), state estimation error which represent the descrepencies between CTSIS (1) and its observer (13) converge exponentially to the nearly of zero, if the term $\beta > 0$, there exist matrices $P_1 > 0$, $P_2 > 0$, Q and G_j $j = 1, \ldots, r$ as well as a positive scalar θ, checking the considered LMIs constraints:

$$\Sigma_{ij} = \begin{pmatrix} \Sigma_{11} & * & * \\ \Sigma_{21} & \Sigma_{22} & * \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{pmatrix} < 0 \quad \forall(i,j) \in \{1,\ldots,r\} \tag{31}$$

where

$$\left\{\begin{array}{lcl} \Sigma_{11} & = & \tilde{A}_j^T\Omega_1^T P_1 + \tilde{A}_j^T\Omega_3^T Q^T - \tilde{C}^T G_j^T + 2\beta P_1 + P_1\Omega_1^T\tilde{A}_j + Q\Omega_3^T\tilde{A}_j - G_j\tilde{C} + I \\ \Sigma_{21} & = & \Delta\tilde{A}_{ij}\Omega_1^T P_1 + \Delta\tilde{A}_{ij}\Omega_3^T Q^T \\ \Sigma_{22} & = & 2\beta E^T P_2 E + \tilde{A}_i^T P_2 E + E^T P_2\tilde{A}_i \\ \Sigma_{31} & = & \Delta\tilde{B}_{ij}^T\Omega_1^T P_1 + \Delta\tilde{B}_{ij}^T\Omega_3^T Q^T \\ \Sigma_{32} & = & \tilde{B}_i^T P_2 E \\ \Sigma_{33} & = & -\theta I \end{array}\right. \tag{32}$$

The fuzzy observer gains are given by:

$$\left\{\begin{array}{lcl} R & = & P_1^{-1}Q \\ \tilde{H}_j & = & (\Omega_1 + R\Omega_3)\tilde{B}_j \\ J_{2j} & = & P_1^{-1}G_j \\ M_j & = & (\Omega_1 + R\Omega_3)\tilde{A}_j - J_{2j}\tilde{C} \\ J_{1j} & = & M_j(\Omega_2 + R\Omega_4) \end{array}\right. \tag{33}$$

Where Ω_1, Ω_2, Ω_3, and Ω_4 are such that (12) is satisfied.
Proof of Theorem 1: The quadratic Lyapunov function is introduced :

$$V = (\bar{E}\bar{\varepsilon})^T P(\bar{E}\bar{\varepsilon}), \qquad P > 0 \tag{34}$$

with

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \tag{35}$$

The time derivative of V along the trajectory of (29) is described by:

$$\dot{V} = \dot{\bar{\varepsilon}}^T\bar{E}^T P\bar{E}\bar{\varepsilon} + \bar{\varepsilon}^T\bar{E}^T P\bar{E}\dot{\bar{\varepsilon}} \tag{36}$$

It follows that the equality (36) is developed to:

$$\dot{V} = \sum_{i,j=1}^{r}\lambda_i(\eta)\lambda_j(\hat{\eta})((\bar{\varepsilon}^T\Pi_{ij}^T + u^T\Lambda_{ij}^T)P\bar{E}\bar{\varepsilon} + \bar{\varepsilon}^T\bar{E}^T P(\Pi_{ij}\bar{\varepsilon} + \Lambda_{ij}u)) \tag{37}$$

Taking into account (34), (37) becomes:

$$\dot{V} = \sum_{i,j=1}^{r}\lambda_i(\eta)\lambda_j(\hat{\eta})(\bar{\varepsilon}^T(\Pi_{ij}^T P\bar{E} + \bar{E}^T P\Pi_{ij})\bar{\varepsilon} + u^T(\Lambda_{ij}^T P\bar{E})\bar{\varepsilon} + \bar{\varepsilon}^T(\bar{E}^T P\Lambda_{ij})u) \tag{38}$$

The stability of (29) and the bounded transfer from u to ε: is assured as:

$$\frac{\|\bar{\varepsilon}\|_2}{\|u\|_2} < \tau \quad , \qquad \|u\|_2 \neq 0 \tag{39}$$

Exponential convergence of error estimation is guaranteed as :

$$\dot{V} + \bar{\varepsilon}^T\bar{\varepsilon} - \tau^2 u^T u < -2\beta V, \quad \beta > 0 \tag{40}$$

From (34) and (38), inequality (40) in compact form can be stated as:

$$\sum_{i,j=1}^{r}\lambda_i(\eta)\lambda_j(\hat{\eta})\left(\bar{\varepsilon}^T u^T\right)\Sigma_{ij}\begin{pmatrix}\bar{\varepsilon}\\ u\end{pmatrix} < 0 \tag{41}$$

where

$$\Sigma_{ij} = \begin{pmatrix} \Pi_{ij}^T P\bar{E} + \bar{E}^T P\Pi_{ij} + S^T S + 2\beta\bar{E}^T P\bar{E} & * \\ \Lambda_{ij}^T P\bar{E} & -\tau^2 I \end{pmatrix} < 0 \tag{42}$$

Thus, from (42), the LMI conditions (31) of Theorem 1 can be established by using (27), (30) and the change of variables:

$$\left\{\begin{array}{lcl} Q & = & P_1 R \\ G_j & = & P_1 J_{2j} \quad \forall\, j \in \{1, \ldots, r\} \\ \theta & = & \tau^2 \end{array}\right. \tag{43}$$

Finally, after solving the LMI conditions (31), the observer gains (33) can be calculated from (20), (23), (24) and (43).

4. SIMULATION RESULTS

In this part the studied CTSIS with actuator and sensor fault , in case of unmeasurable premise variable is written as:

$$\left\{\begin{array}{lcl} \Gamma\dot{\xi} & = & \sum_{j=1}^{2} \lambda_j(\eta)(A_j\xi + B_j u + \Lambda_{aj}\gamma_a) \\ y & = & C\xi + Du + \Omega_a\gamma_a + \Lambda_s\gamma_s \end{array}\right. \tag{44}$$

where $\xi = [\xi_1, \xi_2, \xi_3, \xi_4] \in R^4$ is the state vector, $u \in R$ known input, $y \in R^2$ is the output measurement vector, $\gamma_a \in R$ is the actuator fault and $\gamma_s \in R$ is the sensor fault. The numerical values of the matrices are from [29]:

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2.5 & -0.75 & 0 & 0.025 \\ 0 & 1 & -0.4 & 0 \\ -2.5 & -0.75 & 0 & 0.07 \end{pmatrix}; A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2.696 & -0.75 & 0 & 0.025 \\ 0 & 1 & -0.4 & 0 \\ -2.696 & -0.75 & 0 & 0.07 \end{pmatrix}; D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \; ; \; B_j = \Lambda_{aj} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -0.125 \end{pmatrix} \; ; \; \Lambda_s = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \; C = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

The weighting functions are given by:

$\lambda_1(\eta) = 6.39 + 5.81\xi_1^2 \;\; ; \;\; \lambda_2(\eta) = 1 - \lambda_1(\eta)$

We use the following initial conditions Simulation:

$\xi_0 = [0.05 \;\; 0.35 \;\; 0.75 \;\; -0.67]^T \quad , \quad \hat{\xi}_0 = [0.03 \;\; 0.38 \;\; 0.77 \;\; -0.70]^T.$

Considering the studied simulation results with the use of assumption 1, the unknown fault signal are noted γ_a, γ_s, these signals and their derivatives exercised in time period [30, 245s], as well as [300, 505s], respectively. Subsequently, the application of the devoted fuzzy observer (13) for CTSIS (44), impose the translation of the model (44) in its duplicate structure (10) as recalled in section 2. The aforementioned hypothesis H_1), H_2) also H_3) are assured and under hypothesis H_3) the matrices Ω_1, Ω_2, Ω_3, Ω_4 satisfying (7) are calculated. The earned LMIs (31) in P_1, P_2, Q and G_j was accomplished by taking $\beta = 0.60$ and the bound of norm $\mathcal{L}_2$ is fixed at 0.000195. The input data change during the three time interval, it can be clearly seen that there is no impact on the quality of estimation, since the discrepancies between the real states, faults and their estimated are nearly null, in other word the estimation error are very weak, then states estimation, actuator and sensor fault estimation, likewise stability of the system are well achieved and the mission of the new synthesized observer are accomplished. To confirmed the accuracy of this result from the Figures 1 and 2 we can appreciate that the observer give an exact values of state and faults close to the studied systems the results illustrated in figures reveal that in both of states and faults T-S systems and observers trajectories have similar form. For the representation of the figures, the first Figure 1 ilustrate the trajectories of the states of a rolling disc system [29] with the four states represented respectively by the position of the center of this disc, the translational velocity of the same point, the angular velocity of the disc, and the contact force between the disc and the surface. The second Figure 2 show the applied faults which attack the studied model of the rolling disc system, in this cas there are an actuator and sensor fault in addition to their derivatives.

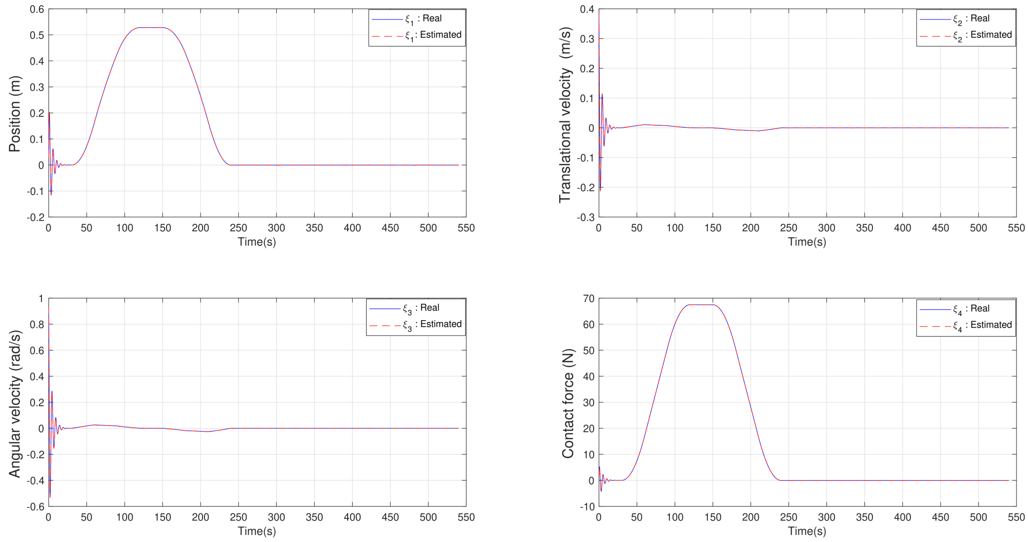

Figure 1. States with their estimates

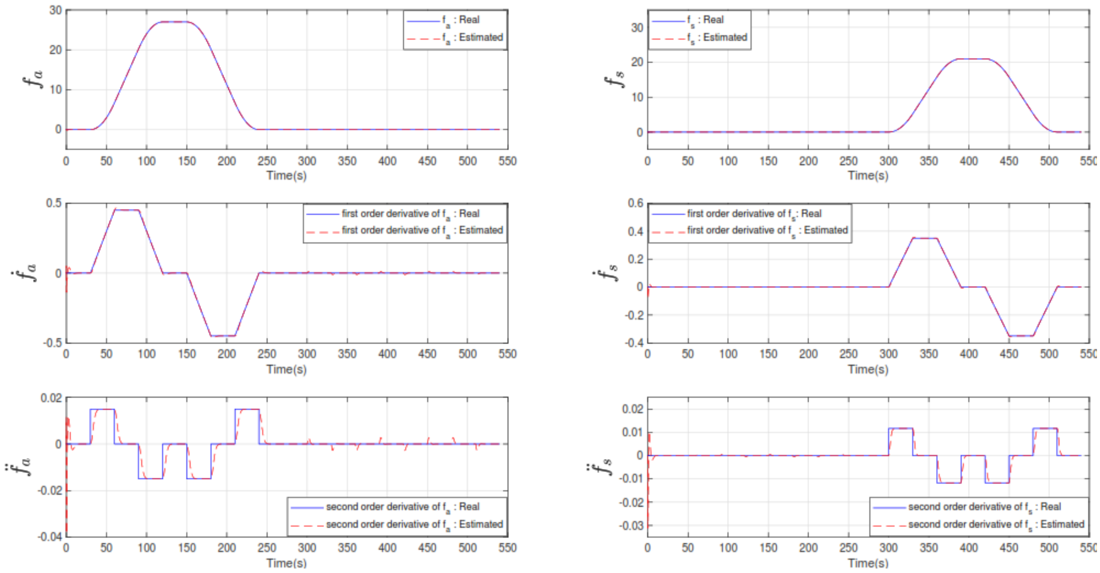

Figure 2. Unknown faults and their derivatives, with their estimates

5. CONCLUSION

In this paper, a new method is proposed to design a fuzzy observer for CTSISs with unmeasurable premise variable. The structure of this observer is inspired by SVD approach as well as an augmented system. This observer is capable to estimate both of state and faults in case of unmeasurable premise variables. The exponential convergence conditions of the observer are studied by a Lyapunov quadratic candidate function and solved via LMIs technique. In this way, the potential and accuracy of the suggeseted method are verified and validated by a numerical example.

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