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An adaptive metaheuristic approach for risk-budgeted portfolio optimization

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ABSTRACT

An investment portfolio implies the assortment of assets invested in the commodity market and equity funds across global markets. The critical issue associated with any portfolio under its optimization entails the achievement of an optimal Sharpe ratio related to risk-return. This issue turns complex when risk budgeting and other investor preferential constraints are weighed in, rendering it difficult for direct solving via conventional approaches. As such, this present study proposes a novel technique that addresses the problem of constrained risk budgeted optimization with multiple crossovers (binomial, exponential & arithmetic) together with the hall of fame (HF) via differential evolution (DE) strategies. The proposed automated solution facilitates portfolio managers to adopt the best possible portfolio that yields the most lucrative returns. In addition, the outcome coherence is verified by monitoring the best blend of evolution strategies. As a result, imminent outcomes were selected based on the best mixture of portfolio returns and Sharpe ratio. The monthly stock prices of Nifty50 were included in this study.

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1. INTRODUCTION

A meta-heuristic framework, namely differential evolution hall of fame (DEHOF), was deployed by Pai and Michel [1] to resolve the issue of equity market neutral portfolio (EMNP) with specific bounding constraints imposed on long-short positions and high-risk assets with a penalty system to manage intricate forces for maintaining excellent portfolios within a limited time. Meanwhile, Wang and Hu [2] prescribed a set of evolutionary algorithms to generate the best outcomes for variance and co-variance optimization frameworks. Alternatively, sinusoidal differential evolution (Sin DE) has been introduced by Draa *et al*. [3]. The recommended approach is studied against the linear parameter shifting DE, the classical model of DE. Kamili and Riffi [4] analyzed the application of metaheuristics in portfolio optimization. It comprises particle swarm optimization (PSO), bat algorithm (BA), and cat swarm optimization (CSO). Zaheer *et al*. [5] have considered the mean semi-variance portfolio optimization model given by Markovitz, solved with the help of DE, which is a population-based metaheuristic. Jia and Bai [6] discussed an uncertain portfolio selection problem considering background risk and asset liquidity.

The role of DE in electricity markets for resolving the issue of portfolio optimization was elaborated by Faia *et al*. [7]. Smart grid technologies were discussed by Lezama *et al*. [8] to integrate and manage the distributed energy sources. Meanwhile, Sethia [9] compared performance and convergence time of varying swarm-intelligence-based methods for two types of objective functions. The objective functions refer to Sharpe ratio maximization and maximization of value at risk-weighted portfolio return. Cortés *et al.* [10] built a dynamic portfolio to identify the most efficient parameters of the PSO framework by employing artificial intelligence that applies PSO in addition to genetic algorithm (GA).

A new decision-making approach regarding multiple risks involved in portfolio optimization was initiated by Salehpoor and Zavardehi [11] by endorsing cardinality restraints, which are referred to as hybrid or mixed meta-heuristic algorithms. Next, Fernandez *et al.* [12] proposed a model of time-related effects under imperfect knowledge and its impact on selecting optimal new product development portfolios. Hu *et al.* [13] conducted comparative experiments on multi-swarm multi-objective optimization evolutionary algorithms based on p-optimality criteria (p-MSMOEAs), while several multiple objective evolutionary algorithms (MOEAs) were evaluated based on six mathematical benchmarking functions with two portfolio samples. A linear portfolio selection method was initiated by Khan *et al.* [14] by weighing in the cardinality inhibitors and the cost of transaction. Next, Costa and Kwon [15] proposed an approach that relaxed the generalized risk parity model into a convex semidefinite program.

As opined by Harrison *et al*. [16], in the case of a defined set of projects, selecting and scheduling the optimum subset of projects are challenging issues recognized as nondeterministic polynomial time (NP)-Hard and addressed as project portfolio selection and scheduling problem. It was demonstrated by Kalayci *et al*. [17] that an effective mixed metaheuristic algorithm blends the significant elements of an artificial bee colony (ABC), consistent optimization of an ant colony, and GAs to resolve the issue of cardinality restricted portfolio optimization. Escobar-Anel *et al*. [18] prescribed two approaches: First, the optimization problem is reduced to an associate problem with constraints independent of wealth and a different utility function. Second, a change of control is applied. Following the 2008 financial crisis in the US, the equal risk contribution of the asset allocation model was developed by Davallou *et al*. [19], in which special attention as a risk factor. Another metaheuristic, adaptive multi-population optimization, was studied by Li *et al*. [20] to yield consistent optimization. Higher entropy values generate higher portfolio diversifications, which can minimize portfolio risk. As such, Lam *et al*. [21] introduced a multi-objective optimization model, namely a mean-absolute deviation-entropy model. Meanwhile, Meng *et al*. [22] revisited the bi-criteria portfolio optimization model with permissible short selling. Next, Cura [23] developed a heuristic approach to the portfolio optimization problem by using the ABC technique.

Both GA and PSO have been commonly deployed as issue-solving methodologies for risk budgeting optimization. Metaheuristic optimization with DEHOF can address a range of quality implications linked with risk-budgeted portfolio optimization. Turning to this present study, a novel technique is proposed to overcome the problem of constrained risk-budgeted optimization with multiple crossovers along with DEHOF. This proposed automated solution helps portfolio managers to adopt the best possible portfolio that yields the greatest return. The study outcomes may guide risk management and investors to select the most viable asset for optimum returns. The rest of this paper is described: i) The theoretical background is described in detail in section 2 and explains the proposed solution, an adaptive metaheuristic approach for risk-budgeted portfolio optimization; ii) Section 3, results, and discussion of its implementation by selecting the Nifty50 portfolio was presented; and iii) Section 4, the conclusion of this study is offered.

2. THEORITICAL BACKGROUND–RISK-BUDGETED PORTFOLIO OPTIMIZATION

2.1. Risk-budgeted portfolio optimization model

The risk budgeting investment, commonly referred to as risk contribution, implies a technique of value investment that enhances the exposure to investment and market protection through the segregation of aggregate risks of the portfolio into component risks. Consequently, wealth distribution entails harmonizing with risk budgets based on the risk-taking capacity of the investors. Risk budgeting could be effected through what is known as marginal contribution to risk (MCR). MCR is defined as the partial derivative of the portfolio risk with respect to its weights and is given by,

$$\bar{m} = (m_1, m_2, \dots, m_n)' = \frac{V.\bar{w}}{\sqrt{\overline{w'}.V.\bar{w}}} \quad (1)$$

$\overline{w}' = (w_1, w_2, \dots, w_N)$ implies the weight set, whereas V defines the variance-covariance matrix of asset returns. The absolute contribution to total risk is given by,

$$\bar{w}_i.m_i, i = 1,2,3, \dots N, \quad (2)$$

and the percentage contribution to total risk is given by,

$$\frac{Absolute\ Contribution\ to\ Total\ Risk}{Total\ Risk} = \frac{w_i.m_i}{\sqrt{w'.V.\bar{w}}}, i = 1,2,\ldots,N \quad (3)$$

It is an investor's prerogative to stretch the risk budgeted portfolio to the maximum limit by investing in an advantageous long-term fund and optimizing the Sharpe ratio of the portfolio by constraints (4), a risk budget of r% is imposed over the absolute contribution to total risk for each asset i in the portfolio, that is,

$$w_i.m_i \leq r\% \quad of \quad \sigma_P, i = 1,2,\ldots,N \quad (4)$$

where m_i is the marginal contribution to the risk linked with the asset i, w_i implies the weight of asset i, and σ_p the portfolio's risk. The portfolio is fully invested, that is,

$$\sum_{i=1}^{N} W_i = 1 \quad (5)$$

unbounded inequality constraints are imposed on specific assets, especially the ones with the positive premium, which indicates the extent of leveraging permitted to the portfolio, that is,

$$w_i > 0 \ \ or \ \ w_j \geq 0, \ i \neq j \quad (6)$$

a constraint enforced on selected assets for defining the leveraging limit of a set of the long-short portfolio, that is,

$$-a_i \leq w_i \leq b_i \quad (7)$$

where $(a_i, \ b_i)$ are upper and lower limits accepted by the investor to boost a leveraged portfolio.

Thus the risk-budgeted portfolio which is a leveraged long-short portfolio comprises three asset classes with specific constraints imposed on each of these asset classes. Some are positive premia holding compulsory and beneficial investment ($w_i > 0$), other positive premia assets with optional and leveraged investment ($w_j > 0$), other assets with free bounds but leveraged and long-short ($-a_i \leq w_i \leq b_i$). Let W^+, W^{Spl}, and W^{Free} reflect the three categories of assets. The mathematical formulation of the problem model is described as,

$$max\left(\frac{\bar{p}.\bar{w}}{\sqrt{w'.V.\bar{w}}}\right) \qquad (maximize\ Sharpe\ Ratio) \quad (8)$$

where $\bar{p}$ corresponds with the premia (returns) of the assets in the portfolio, $\bar{w}$ implies the weights and V means variance-covariance matrix of asset returns, $\bar{p}\bar{w}$ defines the expected portfolio return, $\sigma_P = \sqrt{w'.V.\bar{w}}$ symbolizes the portfolio risk and $\frac{\bar{p}.\bar{w}}{\sqrt{w'.V.\bar{w}}}$ computes the Sharpe ratio, subject to the constraints,

$$w_i.m_i \leq r\% \ \ of \ \ \sigma_p, i = 1,2,\ldots,N \qquad (risk \ \ budgeting\ ) \quad (9)$$

where $\bar{m} = (m_1, m_2, \ldots, m_N)' = \frac{V.\bar{w}}{\sqrt{w'.V.\bar{w}}}$ indicate the marginal contributions to risk and $r\%$ is the limit of the risk,

$$\sum_{i=1}^{N} = 1 \qquad (fully \ \ invested) \quad (10)$$

$$w_j^+ > 0 \quad (financial \ \ leverage) \quad (11)$$

where w^+ imply the weights of selected assets of positive premia W^+,

$$w_k^{spl} \geq 0, \ \ j \neq k \ (optional \ \ inclusion \ \ of \ \ special \ \ leveraged \ \ asset) \quad (12)$$

where w_k^{spl} define the weights of specific assets W^{Spl} whose inclusion is not compulsory but, if inclusive, can be leveraged to an unlimited extent.

$$-a_i \leq w_i^{Free} \leq b_i \quad (long - short\ mix,\ promoting\ leveraging) \tag{13}$$

where $(-a_i,\ b_i)$ are free limits for $a_i,\ b_i$ accepted by the investor for selected assets belonging to W^{Free} and are promoting a beneficial long-short portfolio. In (8)–(13) define a single objective non-linear constrained fractional programming model which is difficult to solve using analytical methods and hence the need for metaheuristic methods. However, to tackle the non-linear constraints represented by (9) together with the linear constraints (bounded and unbounded) represented by (10)–(13), it is essential that metaheuristic methods adopt specialized methods which may involve transformation of the original problem model. The constraint management strategies implemented under metaheuristic techniques are given below.

2.2. Constraint management

Considering the integration of combined metaheuristic models, one of the key barriers is the need to overpower the restrictions placed on the problems. Therefore, metaheuristic models must generate feasible solutions to comply with each paradigm restriction before attaining an appropriate solution. Thus, the literary studies focus on multiple approaches for mitigating the issue of constraint management.

2.3. Strategy of repair

The set of chromosomes or individuals having genes or gene components reflecting the individual's set of optimization solutions has been examined to ascertain if they represent feasible sets of problems. In case they violate either of the defined restrictions, they are termed as "infeasible". The well-structured repair techniques pursue repairing the infeasible individuals or chromosomes to transform them into feasible sets of solutions. However, the fact that no standard repair technique is likely applicable for all issues is quite complex makes it disadvantageous. It can hamper the process of evolution of problem solutions.

2.4. Strategy of penalty function

The penalty function strategy is based on similar methods adopted in conventional constrained optimization, where solutions that are infeasible are penalized using what are called penalty coefficients. The strategy of penalty function focuses on comparative approaches launched in traditional constrained optimization; wherein infeasible techniques are subjected to penalty through the usage of penalty coefficients. Thus, given a constrained optimization problem, which is a minimization problem as described as,

$$\begin{gathered} min(f(\bar{x})),\ \bar{x} = (x_1, x_2, \ldots, x_n) \\ g_k(\bar{x})) \leq 0,\ k = 1,2,\ldots,K) \\ h_m(\bar{x})) = 0,\ m = 1,2,\ldots,M) \\ x_i \in dom(x_i) \end{gathered} \tag{14}$$

where $\bar{x} = (x_1, x_2, \ldots, x_n)$ are the decision variables with $dom(x_i)$ as the domain of the decision variable x_i, $g(\bar{x})$ reflect the inequality constraints, $f(\bar{x})$ implies the minimization objective function and $h_m(\bar{x})$ are the equality constraints. The penalty function strategy transforms the constrained optimization problem model into an unconstrained optimization problem model by reconstructing the objective function as,

$$\phi(\bar{x}, r) = f(\bar{x}) + r\sum_{m=i}^{M} (h_m(\bar{x}))^2 + r\sum_{k=1}^{K} G_k(g_k(\bar{x}))^2 \tag{15}$$

where G_k is the heavier operator such that:

$$G_k = \begin{cases} 0, & g(\bar{x} \leq 0) \\ 1, & g_k(\bar{x}) > 0 \end{cases} \tag{16}$$

and r is a positive magnifier that restricts the magnitude of the penalty function. In the case of a restricted optimization problem that can become an issue of maximization, the change may be influenced by using the principle of duality as,

$$max(f(\bar{x})) = -min(-f(\bar{x})) \tag{17}$$

2.5. Joines and Houck's penalty function method

A penalty-oriented approach to constraint management was proposed by Joines and Houck, which increases the penalty coefficient with the enhancing number of generations, thus explaining the function of the penalized objective in t^{th} generation is described as,

$$\phi(\bar{x},t) = f(\bar{x}) + (C.t)^{\alpha}(\sum_{m=1}^{M} (h_m(\bar{x}))^{\beta} + \sum_{k=1}^{K} G_k(g_k(\bar{x}))^{\beta}) \quad (18)$$

where (C, α, β) are referred to as constants, and penalty term $(C.t)^{\alpha}$ increases consistently with the count of each generation. Nonetheless, the continuity of solutions is based on the selection of (C, α, β) and conclusively, the acceptable substitute to the constants is $(C = 0.5, \alpha = 2, \beta = 2)$. The increasing penalty term of $(C.t)^{\alpha}$ with each generation results in a situation where the infeasible chromosomes / individuals during the last few generations receive death penalty and thus the method tends to converge early.

2.6. Transformed risk budgeted portfolio optimization framework

To tackle the non linear risk-budgeting constraint, Joines and Houck's Penalty function method is employed, which transforms the original objective function (maximization) to its dual (minimization), using the principle of duality. The revised formulations of the objective function and the risk budgeting constraints are as,

$$-min(-\frac{\overline{p}.\overline{w}}{\sqrt{\overline{w'}.V.\overline{w}}} + \psi(\overline{w}, \overline{m}, t)) \quad (19)$$

where $\psi(\overline{w}, \overline{m}, t)$ known as the function of constraint violation is explained as:

$$\psi(\overline{w}, \overline{m}, t) = (C.t)^{\alpha}(\sum_{k=1}^{N} G_k(\varphi_k(w_k, m_k))^{\beta})$$

$\varphi_k(w_k, m_k) = w_k m_k - x\% \ of \ \sigma_P$ and

G_k is the Heaviside Operator such that:

$$G_k = \begin{cases} 0, & for \ \varphi(w_k, m_k) \leq 0, \quad and \\ 1, & for \ \varphi(w_k, m_k) > 0 \end{cases}$$

$$\overline{m} = (m_1, m_2, \ldots, m_N)' = \frac{(V.\overline{w})}{\sqrt{\overline{w'}.V.\overline{w}}} \quad (20)$$

The transformed objective function represented by (19)–(20) together with the linear constraints represented by (10)–(13) now defines the transformed Risk-budgeted portfolio optimization model. The attribute of changed goal with linear constraints elaborates the modified model of risk budgeted portfolio optimization that holds the complex resolving capability of optimizing endless linear constraints, thereby creating metaheuristic methodologies. Furthermore, DEHOF has been identified as a purified variant of the differential evolution algorithm.

2.7. Differential evolution associated with hall of fame

Differential evolution is a form of substitutional development algorithm paired with a hall of fame technique, which is a depository of a couple of best individuals from each generation to promote super specialization. Only the best fit one succeeds in earning a membership. Thus, the hall of fame is extensively witnessed in evolutionary algorithms boosting elitism with time. At the stage of termination of any evolutionary algorithm, especially the differential evolution algorithm, a chromosome or individual occupying space in the hall of fame has declared as the "BEST" solution to the issue [24].

2.8. Repair mechanism for handling unrestricted linear constraints

The repair mechanism is responsible for managing unrestricted linear constraints (11)–(12) subject to restrictions on a stringently dedicated portfolio (10) trailed by restrictions (13) on the free limit of chosen assets to promote the blend of the long-short portfolio having leverage. The objective of a repair strategy in case of the communication between chromosomes or individuals and a pool of random portfolio weights corresponds to fixing or standardizing the weights to satisfy the entire range of direct imperatives communicated with (10)–(13) for transforming it to a pragmatic assortment vector of the problem. Assuming 'W' as an irregular vector of weight regarding portfolio built up by N assets during (-c, +c), W^{+}, W^{Spl}, and W^{Free} stand for the sets of weight-related with the assets falling in three asset categories explained by (11)-(13). The number of assets present in each asset category can be denoted by p, s, and f, such that p+s+f =N. The investors opt for risk budgeted portfolio optimization to achieve a leveraged portfolio of long-short position to maximize their portfolio's Sharpe ratio. Table 1 illustrates the issue constraints as well as criteria that have been decided by the investor, reflecting the preferences of the investor fitting in line with the modified mathematical model of risk budgeted portfolio optimization stipulated by (10)–(13) as linear constraints and as an objective penalty function by (19)–(20).

Table 1. Investor-defined issue parameters and constraints of the portfolio

Parameter	Description	Remarks
Portfolio size	50 assets	
Composition of assets in the portfolio	Automobile: 6	Assets serial No: 4,14,20,32,33,44
	Cement: 3	Assets serial No: 16,40,48
	Construction: 1	Assets serial No: 31
	Consumer Goods: 6	Assets serial No: 2,9,22,28,34,47
	Fertilizer & Pesticides: 1	Assets serial No: 49
	Financial Services: 11	Assets serial No: 3,5,6,18,19,23,24,26,30,39,41
	IT: 5	Assets serial No: 17,27,43,46,50
	Oil & Gas: 5	Assets serial No: 7,15,25,36,38
	Pharma: 4	Assets serial No: 10,12,13,42
	Metal 4	Assets Serial No.11,21,29,45
	Power: 2	Assets serial No: 35, 37
	Services: 1	Assets serial No: 1
	Telecom: 1	Assets serial No: 8
Risk budget	15%	
Portfolio objective	Maximize Sharpe ratio	Equations (19)-(20)
Nature of the portfolio	Leveraged, fully invested, and long short	
Basic constraint	Fully invested portfolio	Equation (10), $\sum_{i=1}^{50} = 1$
Investors defined asset classes and constraints imposed on them	Leveraged weights for nifty50 (Asset serial No: (1-48)	Equation (11) $w_j^{48} > 0$, j $= 1,2,3, \dots 48$
	Mandatory investment	Lower bound set as 0.001
	Leveraged weights for fertilizers & pesticides (Asset serial no: 49)	Equation (12) $w_k^{spl} \geq 0$, k $= 49$
	Optional investment	
	Leveraged long-short weights for Wipro Ltd (Asset serial no.50)	Equation (13) $-a_i \leq w_i^{Free} \leq b_i, i = 50$

Under the attributes of the targeted problem model, DEHOF assesses the health of the population comprising parents and offspring. Considering the risk budget portfolio optimization (19) and (20), the evolutionary objective function is elaborated, which demands penalty functions to mitigate the non-linear risk. Every individual in the population symbolizes the vector of weight. Therefore, fitness values need to be repaired quickly to attain a feasible set of solutions. Marginal contributions to risk can be actuated by substituting w in the targeted individuals with repaired weights, specifying the criteria for penalty functions (C, α, β), the achievement of p and V by t, which imply the premia and variance-covariance metrics of the returns linked with the concerned portfolio, along with calculating m.

The adaptive metaheuristic risk-budgeted portfolio optimization (AMRBPO) algorithm will be explained in the following steps: i) The process of differential evolution with hall of fame initiates with determining problem parameters, identifying assets or asset categories for the portfolio, and defining their lowest limits, setting risk budgets, and achieving premiums and metrics of variance and covariance; ii) This stage is followed by setting DEHOF specifications, determining generations and population size, dynamic penalty functions, setting scale factors and generation index represented as i, and finally, starting the hall of fame; iii) The next step involves the random population generation wherein every gene has an irregular asset weight. After this, the repair strategy is implemented on every individual of the population to transform them into viable solution sets that resolve the constraints; iv) Then, the fitness function values of the population P can be calculated by employing the objective functions. In case the generation index is less than or equal to the population generations, the parent population is established as P, and its fitness values are recorded; v) differential evolution strategies (D.E./rand/1, D.E./rand5/Dir4 & D.E./rand4/BestDir5) and multiple crossovers (Binomial, Exponential & Arithmatic) are applied to this generation for gaining offspring population. The repair strategy applies to all the individuals of the offspring population referred to as O, which transforms the entire generation into a feasible set of solutions; vi) The fitness value of O is calculated before applying the deterministic selection operator and selecting the best individuals from P and O populations for the hall of fame (HF) of the next generation called NEXTGEN; vii) NEXTGEN individual having the best fitness is tagged as BEST and contrasted with the individual in HF, out of which the winner would occupy the final space in HF; viii) The NEXTGEN is retagged as population P, where i = i + 1 and the entire cycle restarts. However, in contrasting situations where i is not equal to or lesser than generations, the weight W of HF individual is identified, which denotes optimal weight. Lastly, the maximum Sharpe ratio is calculated herein; and ix) The optimal blend of all conceivable combinations with the best portfolio returns and Sharpe ratio will be picked.

3. RESULTS AND DISCUSSION

3.1. Data collection

The monthly stock prices of Nifty50 from January 2019 to January 2021 were employed in this study [25] to compute the covariance matrices. The premiums of portfolio Nifty50 achieved by [25] is presented in Table 2. MATLAB 2019a was used for this study.

Table 2. Assets and their premiums comprising the Nifty50 portfolio

S. No.	Description	Premium
1	Adani Port and Special Economic Zone Ltd.	0.006
2	Asian Paints Ltd.	0.021
3	AXIS Bank Ltd.	0.025
4	Bajaj Auto Ltd.	0.007
5	Bajaj Finance Ltd.	0.023
6	Bajaj FinServ Ltd.	0.008
7	Bharat Petroleum Corp. Ltd.	0.005
8	Bharti Airtel Ltd.	0.020
9	Britannia Industries Ltd.	0.007
10	Cipla Ltd.	0.007
11	Coal India Ltd.	0.005
12	Divi's Laboratories Ltd.	0.008
13	Dr. Reddy's Laboratories Ltd.	0.011
14	Eicher Motors Ltd.	0.006
15	GAIL (India) Ltd.	0.004
16	Grasim Industries Ltd.	0.006
17	HCL Technologies Ltd.	0.017
18	HDFC Bank Ltd.	0.104
19	HDFC Life Insurance Co.	0.008
20	Hero MotoCorp Ltd.	0.007
21	Hindalco Industries Ltd.	0.006
22	Hindustan Unilever Ltd.	0.036
23	Housing Development Finance Corporation Ltd.	0.076
24	ICICI Bank Ltd.	0.061
25	Indian Oil Corporation Ltd.	0.004
26	IndusInd Bank Ltd.	0.008
27	Infosys Ltd.	0.083
28	ITC Ltd.	0.030
29	JSW Steel Ltd.	0.006
30	Kotak Mahindra Bank Ltd.	0.049
31	Larsen and Toubro Ltd.	0.026
32	Mahindra and Mahindra Ltd.	0.012
33	Maruti Suzuki India Ltd.	0.017
34	Nestle India Ltd.	0.011
35	NTPC Ltd.	0.008
36	Oil And Natural Gas Corporation Ltd.	0.006
37	Power Grid Corporation of India Ltd.	0.008
38	Reliance Industries Ltd.	0.107
39	SBI Life Insurance Co.	0.006
40	Shree Cement Ltd.	0.005
41	State Bank of India	0.018
42	Sun Pharmaceutical Industries Ltd.	0.011
43	Tata Consultancy Services Ltd.	0.057
44	Tata Motors Ltd.	0.006
45	Tata Steel Ltd.	0.008
46	Tech Mahindra Ltd.	0.010
47	Titan Company Ltd.	0.011
48	UltraTech Cement Ltd.	0.010
49	UPL Ltd.	0.000
50	Wipro Ltd.	0.000

3.2. DE/ rand/1 strategy with multiple crossovers

The execution of the AMRBPO algorithm was done to achieve an optimal portfolio through multiple crossovers with DE/rand/1 strategy. It was clear from Table 3 that the Exponential crossover yields the best returns on the portfolio for the strategy DE/rand/1. This strategy was executed in repetition to examine the consistency of its outcomes. Table 4 displays the outcomes of 10 sample runs, which confirm the consistency of Sharpe ratios solely aiming at portfolio maximization under the DE/rand/1/exp strategy.

3.3. DE/rand 5/Dir4 strategy with multiple crossovers

The execution of the AMRBPO algorithm was done to achieve an optimal portfolio through multiple crossovers with DE/rand5/Dir4 strategy. It was clear from Table 5 that the DE/rand5/Dir4/bin strategy yields the best returns on the portfolio. This strategy was executed in repetition to examine the consistency of its outcomes. Table 6 shows the outcomes of 10 sample runs, which confirm the consistency of Sharpe ratios solely aiming at portfolio maximization under the DE/rand5/Dir4/bin strategy.

Table 3. Risk/returns achieved by AMRBPO algorithm through multiple crossovers

Crossovers	Sharpe ratio	Portfolio risk	Portfolio return
Binomial	0.494	0.073	0.036
Exponential	0.723	0.085	0.062
Arithmatic	0.479	0.088	0.042

Table 4. Risk/return features by DE/rand/1 during multiple cycles of Exponential crossover

Runs	Sharpe ratio	Portfolio risk	Portfolio return
1	0.845	0.111	0.094
2	0.726	0.107	0.067
3	0.980	0.077	0.076
4	0.726	0.107	0.067
5	0.723	0.085	0.061
6	0.723	0.085	0.061
7	0.723	0.085	0.061
8	0.798	0.084	0.067
9	0.756	0.095	0.072
10	0.744	0.104	0.067

Table 5. Risk/return features attained by AMRBPO algorithm through multiple crossovers

Crossovers	Sharpe ratio	Portfolio risk	Portfolio return
Binomial	1.774	0.055	0.097
Exponential	1.739	0.064	0.112
Arithmatic	0.397	0.077	0.031

Table 6. Risk/Return features DE/rand5/dir4 during multiple cycles of Binomial crossover

Runs	Sharpe ratio	Portfolio risk	Portfolio return
1	1.827	0.045	0.045
2	1.774	0.055	0.097
3	1.763	0.054	0.054
4	1.827	0.045	0.045
5	1.735	0.079	0.137
6	1.737	0.075	0.129
7	1.735	0.079	0.137
8	1.737	0.075	0.129
9	1.753	0.046	0.077
10	1.768	0.055	0.055

3.4. DE/rand4/Bestdir5 strategy with multiple crossovers

The execution of the AMRBPO algorithm was done to achieve an optimal portfolio through multiple crossovers with DE/rand4/BestDir5 strategy. It was clear from Table 7 that the best returns on the portfolio are yielded by DE/rand4/ BestDir5/exp strategy, due to which is executed in repetition for many cycles to observe its outcome consistency. Table 8 displays the outcomes of 10 sample runs that reveal the Sharpe ratios consistency, solely aiming to maximize the optimum portfolios under DE/Rand4/BestDir5/exp strategy. This study is a comparative analysis of long-short term metaheuristic risk-budgeted portfolio maximization for the targeted optimization of Sharpe ratios of the assets of the Nifty50 portfolio through the AMRBPO algorithm. Table 9 reveals that the combination of Binomial crossover with DE/rand5/Dir4 yields the highest Sharpe ratio of 1.774 linked with 5.5% risks that yield a return value of 9.7% upon the assets of Nifty50.

Table 7. Risk/returns attained by AMRBPO algorithm with multiple crossovers

Crossovers	Sharpe ratio	Portfolio risk	Portfolio return
Binomial	1.455	0.061	0.089
Exponential	1.633	0.067	0.112
Arithmatic	0.424	0.086	0.036

Table 8 Risk/returns achieved by DE/rand4/Bestdir5 during multiple cycles for Exponential crossover

Runs	Sharpe ratio	Portfolio risk	Portfolio return
1	1.649	0.071	0.111
2	1.657	0.061	0.096
3	1.690	0.058	0.097
4	1.602	0.077	0.077
5	1.625	0.058	0.094
6	1.623	0.059	0.089
7	1.602	0.068	0.109
8	1.600	0.042	0.042
9	1.631	0.050	0.079
10	1.633	0.069	0.112

Table 9. Best risk/returns obtained by AMRBPO with various evolution strategies

Evolution strategy	Best crossover	Sharpe ratio	Portfolio risk	Portfolio return
DE/rand1	Exponential	0.723	0.085	0.062
DE/rand5/Dir4	Binomial	1.774	0.055	0.097
DE/rand4/BestDir5	Exponential	1.633	0.067	0.112

4. CONCLUSION

This work discusses an optimal construction of a risk budgeted portfolio using a meta-heuristic method viz., AMRBPO algorithm. The conclusions are; i) While it is possible to directly solve the optimization problem in its naive form using linear programming techniques but the risk budget constraint imposed on the high-risk assets in the portfolio, including bounding constraints, levied on the long-short positions and high-risk assets with more than two crossovers makes it a complex problem, which is resolved by using AMRBPO algorithm. ii) AMRBPO algorithm reported consistency of performance across all the runs, and iii) The proposed AMRBPO algorithm helps to select the best combination of differential evolution strategy and crossover for the best portfolio returns for the investors.

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