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Optimizing queue efficiency: Artificial intelligence-driven tandem queues with reneging

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ABSTRACT

This paper delves into the theoretical integration of queueing theory and artificial intelligence (AI), examining the benefits and implications of their convergence. Queueing systems serve as fundamental models for various real-world applications, from telecommunications networks to healthcare facilities. This research presents a transformative framework for elevating the efficiency and performance of queueing systems by infusing AI-driven tandem queue analysis. The implications of this approach transcend industries, promising streamlined operations, reduced waiting times, and resource optimization. This work invites further exploration and application, offering a path to more effective and responsive queueing systems globally. Over the years, researchers and practitioners have explored numerous techniques to enhance the efficiency and performance of queueing systems. In recent times, integrating AI into the realm of queueing analysis has opened up new avenues for optimization and innovation. This paper studies a two-server tandem queueing model with reneging customers using AI techniques. Assuming that the arrival rate follows the Poisson process and the service rate follows an exponential distribution, using the birth-death process, probability generating function and AI module, we derive steady-state difference equation, expected number of people in customers, and mean waiting time.

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1. INTRODUCTION

Queueing systems serve as the backbone of numerous industries and operational scenarios, influencing everything from customer satisfaction to resource utilization and cost-effectiveness. As organizations strive to deliver more efficient and responsive services, the integration of artificial intelligence (AI) has emerged as a transformative force. Queueing systems, characterized by their inherent complexity and interdependence of tasks, present a unique set of challenges for optimization. Traditionally, queue theory has provided valuable insights and methodologies for tackling these challenges. However, the advent of AI has introduced a dynamic and data-driven dimension to queueing system optimization, promising unprecedented advancements. Efficient queue management is imperative in diverse fields such as telecommunications, healthcare, and transportation. Tandem queues, comprising interconnected queues, represent a common structure in systems where entities undergo multiple stages of processing or service.

However, the dynamic nature of tandem queues, coupled with the phenomenon of reneging (where customers leave the queue before service), poses significant challenges to traditional optimization methods. This study focuses on harnessing the capabilities of AI to analyze and optimize tandem queues, particularly in the context of reneging behavior. The main objectives are:

– Develop AI models: create AI algorithms capable of predicting and managing customer behavior, particularly reneging, in tandem queue systems.
– Optimize queue operations: design and implement strategies to minimize the negative impacts of reneging on service efficiency and customer satisfaction.
– Simulation and validation: conduct simulations to compare the performance of AI-driven queue management against traditional methods.
– Practical implementation: propose actionable insights and frameworks for integrating AI-driven queue management solutions in real-world applications.

Tandem queue systems are commonly used in various service environments, such as banks, hospitals, and call centers. However, customer reneging poses significant challenges, leading to inefficiencies and reduced customer satisfaction. Traditional queue management strategies often fail to address the dynamic and unpredictable nature of customer behavior effectively. This research seeks to address the problem of optimizing tandem queue systems by leveraging AI to predict and manage reneging, thus improving overall efficiency and service quality. The importance of optimizing queue efficiency with AI-driven tandem queues and addressing reneging:

– Customer satisfaction: efficient queue management reduces wait times and uncertainty, enhancing overall customer experience and satisfaction.
– Operational efficiency: AI optimizes resource allocation and staffing, minimizing costs and maximizing throughput.
– Cost savings: effective management reduces operational expenses by aligning staffing levels with demand, avoiding overstaffing or underutilization.
– Business performance: improved efficiency boosts revenue generation and customer retention, providing a competitive edge.
– Adaptability: AI adapts to varying demands and disruptions, maintaining performance under dynamic conditions.

Adepoju [1] investigates the behavior of tandem queues within the context of porous mediums. Wang *et al*. [2] develop models to predict queue behavior and analyze various strategies to mitigate the negative effects of impatience. Alfred [3] paper investigates scenarios where queues within a network can become blocked and items may withdraw from the queue. Ayyappan and Thamizhselvi [4] focus on the transient behavior of queueing systems that incorporate priority services. Yuan *et al.* [5] investigated the potential of AI in optimizing network association for next-generation mobile networks, achieving enhanced QoS and network performance. Ferencz and Zöldy [6] explored the application of AI methods in estimating road traffic queue lengths, showcasing promising results for intelligent transportation systems. The study by Anussornnitisarn and Limlawan [7] demonstrated the potential of artificial neural network (ANN)-based waiting time prediction in optimizing queue management, providing valuable insights for service industries seeking to minimize customer wait times. Kamoun's [8] findings are significant for designing and optimizing queueing systems in environments where prioritization is crucial, such as in the healthcare and customer service sectors. Liu and Quan [9] discuss the synergistic relationship between queueing theory and AI in their paper in [10] explore the synergistic relationship between queueing theory and AI. This study highlights the practical benefits of incorporating machine learning into queue management, particularly in digital communication and data processing environments.

Ayyappan and Thilagavathy [11] examine the application of machine learning techniques to analyze complex queueing systems. Mas *et al*. [12] highlight various AI techniques, such as machine learning and neural networks, to predict wait times, improve service efficiency, and enhance overall system performance. Li *et al*. [13] review the intersection of queueing theory and AI in the context of service systems. This comprehensive survey covers the theoretical foundations and practical applications of AI-enhanced queueing models, demonstrating how AI can be used to address common challenges in service systems, such as variability in service demand and resource allocation. Li *et al.* [13] offer valuable insights for managing queueing systems in environments where space and patience are limited, such as in telecommunications and transportation. Shekhar *et al.* [14] findings offer valuable insights for managing queueing systems in environments where space and patience are limited, such as in telecommunications and transportation. Baghel and Jain [15] work is particularly relevant for designing robust systems in critical applications such as aerospace, healthcare, and information technology (IT) infrastructure.

The research presented in [16]–[18] serves as a foundation for grasping the fundamental role of queueing theory in modeling and analyzing various everyday systems and processes. Ndiaye *et al*. [17] development of AI-driven queue management systems showcases the potential for advanced technologies to enhance efficiency and customer experience. Meanwhile, Stintzing and Norrman's [18] exploration of predictive modeling with ANNs offers a glimpse into the future of dynamic and intelligent queue management. Yuvarani and Vijayalakshmi [19] explore the use of stochastic processes in designing queueing models with ANNs. The authors in [20], [21] propose a queue network model using Kendall's notation to minimize intersection waiting time in their paper. Anuruddhika *et al*. [22] provide a comprehensive overview of the different methodologies and techniques used in queueing modeling, emphasizing the evolution and advancements in the field over time. The authors in [23], [24] explores how queueing models can be applied to optimize resource allocation, demonstrating the relevance of queueing theory in emerging technologies and analyzed multi-server tandem queue with Markovian arrival process, phase-type service times, and finite buffers. Niranjan *et al*. [25] used an optimum cost analysis to minimize the total average cost with data transmission and data processing in long term evolution-advanced (LTE-A) networks using the discontinuous reception (DRX) mechanism.

There is limited research on how AI-driven queue management impacts overall customer experience, particularly in scenarios involving reneging. Studies that evaluate customer satisfaction and perceived fairness in AI-optimized queueing systems are needed to ensure that technological advancements align with customer expectations. Addressing these gaps through focused research can lead to more efficient, scalable, and customer-friendly queue management systems, leveraging the full potential of AI in tandem queue scenarios with reneging. This study investigated the effects of AI-driven optimization on tandem queues with reneging behavior. While earlier studies have explored the impact of traditional queue management techniques and static optimization on queue performance, they have not explicitly addressed the influence of real-time, AI-driven dynamic adaptation on the efficiency and stability of tandem queue systems, especially in the context of customer reneging.

The paper is structured as follows: section 2 discusses the interaction between tandem queueing theory and the field of AI. Section 3 outlines the methods used and provides analysis. Section 4 presents a numerical illustration. Finally, section 5 offers conclusions.

2. INTERACTION BETWEEN THE TANDEM QUEUEING THEORY AND THE AI FIELD

The integration of tandem queues and AI can revolutionize operations research by optimizing queueing systems and enhancing decision-making processes. AI-powered algorithms can analyze and simulate tandem queue operations, leading to improved efficiency and reduced wait times. By leveraging machine learning and queueing theory, researchers and practitioners can develop innovative solutions for complex queueing systems.

2.1. Queue management

AI can be used to optimize the management of tandem queues in various systems, such as customer service centers, manufacturing processes, and transportation systems. AI algorithms can predict queue lengths and arrival rates, helping to allocate resources efficiently and minimize wait times. Predictive maintenance: in manufacturing settings where tandem queues are common, AI can be utilized for predictive maintenance. By analyzing data from sensors and production processes, AI algorithms can anticipate equipment failures or bottlenecks in the queue and schedule maintenance proactively, reducing downtime and improving overall efficiency.

2.2. Dynamic resource allocation

AI algorithms can dynamically allocate resources within tandem queues based on real-time data. For example, in a call center, AI can analyze customer inquiries and assign them to the most appropriate agent based on factors such as skill level, availability, and past performance techniques. AI techniques such as reinforcement learning can be applied to optimize the performance of tandem queues. By continuously learning from past experiences and feedback, AI algorithms can adaptively adjust queueing strategies to improve throughput, reduce congestion, and enhance overall system efficiency.

2.3. Simulation and modeling

AI can enhance the accuracy of tandem queue simulations by leveraging machine learning algorithms to analyze historical data. These algorithms can identify patterns and trends, enabling the development of more realistic simulation models. By incorporating these insights, researchers can optimize queueing parameters and better predict system behavior, leading to improved performance and efficiency.

2.4. Dynamic pricing and queue management

In service industries like airlines or theme parks, AI can help optimize pricing strategies based on demand and queue length. By analyzing historical data and current demand trends, AI algorithms can dynamically adjust prices to balance supply and demand and minimize queue congestion. The novelty of integrating tandem queues with AI lies in the ability to harness advanced computational techniques to tackle complex queueing problems in innovative ways. Here are some aspects contributing to the novelty.

2.5. Personalized queue management

AI-powered queue management can be tailored to individual preferences, taking into account historical behavior and other relevant factors. This personalized approach enables a more efficient and satisfying customer experience. By deviating from traditional one-size-fits-all methods, businesses can significantly enhance customer satisfaction and loyalty.

2.6. Integration with internet of things and sensor data

AI-powered queue management systems can harness real-time data from internet of things (IoT) devices and sensors to gain a deeper understanding of queue dynamics. This integration enables more precise monitoring and control of queueing processes, facilitating data-driven decision-making. By leveraging IoT data, businesses can respond more effectively to changing queue conditions, optimizing the customer experience.

2.7. Simulated environments for experimentation

AI enables the creation of simulated environments for testing and optimizing queueing systems, allowing for experimentation with various strategies. This virtual testing ground facilitates rapid iteration and improvement without disrupting real-world operations. By leveraging AI-driven simulation, businesses can identify optimal queueing solutions and implement them more efficiently.

2.8. Novel approaches of artificial intelligence techniques

Novel approaches to queue management often involve combining multiple AI techniques, such as reinforcement learning and deep learning. By leveraging the strengths of each technique, researchers can develop more robust and effective solutions that address various aspects of queue management. This multi-faceted approach enables the creation of sophisticated queue management systems that can adapt to changing conditions.

2.9. The significance of the field

Theservice industry has undergone a transformative shift since the digital revolution, with automation replacing traditional customer support methods. Leveraging AI and consumer data, businesses can now optimize operations, reduce costs, and enhance customer satisfaction. Integrating tandem queues with AI introduces innovative queue management techniques, elevating efficiency and operational performance across industries such as marketing, e-commerce, sales, and data science. By harnessing refined consumer data, companies can unlock AI's full potential, revolutionizing customer interactions and driving growth through streamlined, sophisticated operations.

3. MODEL DESCRIPTION

In this section, we consider a two-terminal system where customers require services at both ends. Customers enter the system at S_1, queue for one service as they exit, and then go to the second service to use it. The AI module sites between the queues and serves as the intelligent processing layer, Figure 1 represent the AI-driven tandem queueing model. This diagram illustrates a tandem queue system optimized by AI. Arrival customers first enter server 1, where a portion of them may renege and leave the queue, represented by (1−θ) λ. The AI system dynamically manages the flow between server 1 and server 2 to optimize overall queue efficiency. Customers successfully processed by server 1, at a rate β_1, are then directed to server 2, which processes them at a rate β_2, leading to their eventual departure from the system.

3.1. Notations

AI module: χ-arrival rate at server S_1; β_1-service rate at server S_1; β_2-service rate at server S_2; K_1-Number of clients on server S_1. K_2-Number of clients on serverS_2. θ-Reneging client from server S_1. $\zeta_1 = \frac{\chi}{\beta_1}$, $\zeta_2 = \frac{\chi}{\beta_2}$=traffic density.

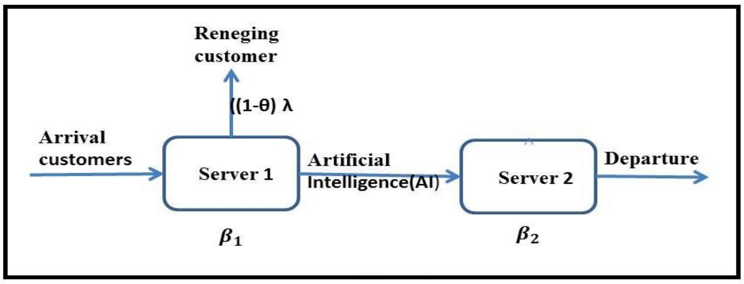

Figure 1. AI-driven tandem queueing model

Incorporating AI to determine or optimize the time t+h in a birth-death process is an interesting application that can help adapt the model's time resolution.

- K_1, K_2 units in the system at time t and no arrival and no service during t+h in both service stations
- $K_1 - 1, K_2$ units in the system at time t, one arrival and no service at server 1, no arrival and no service at server 2 during t+h
- $K_1 + 1, K_2 - 1$ units in the system at time t, no arrival and one service at station 1, one arrival and no service at station 2 during t+h
- $K_1, K_2 + 1$ units in the system at time t, no arrival and no service at server 1, no arrival and one service at server 2 during t+h
- $K_1 + 1, K_2$ units in the system at time t, no arrival and one service at server 1, no arrival and no service at server 2 during t+h

$$\begin{aligned}
&Q_{K_1,K_2}(t+h) = Q_{K_1,k_2}(t)(1-\chi h)(1-\theta\beta_1 h)(1-\beta_1(1-\theta))(1-\beta_2 h) \\
&+Q_{k_1-1,k_2}(t)\chi h(1-\theta\beta_1 h)(1-\beta_1(1-\theta))(1-\beta_2 h) \\
&\theta\beta_1 h Q_{k_1+1,k_2-1}(t)(1-\lambda h)(1-\beta_2 h) + (1-\theta)\beta_1 h P_{K_1+1,K_2}(t)(1-\lambda h)(1-\beta_2 h) \\
&+P_{K_1,K_2+1}(t)(1-\lambda h)(1-\theta\beta_1 h)(1-\beta_1(1-\theta))\beta_2 h \qquad (1)
\end{aligned}$$

Simplifying (1) gives:

$$\begin{aligned}
&Q_{K_1,K_2}(t+h) = P_{K_1,K_2}(1-\beta_1-\beta_2-\chi)\Delta t + \chi h P_{K_1-1,K_2} \\
&+\theta\beta_1 h Q_{K_1+1,K_2-1} + (1-\theta)\beta_1 h P_{K_1+1,K_2} + \beta_2 h P_{K_1,K_2+1} \qquad (2)
\end{aligned}$$

$$\begin{aligned}
\lim_{h\to 0} \frac{Q_{K_1,K_2}(t+h) - P_{K_1,K_2}(t)}{h} &= -Q_{K_1,K_2}(\beta_1+\beta_2+\chi) + \chi Q_{K_1-1,K_2} + \theta\beta_1 Q_{K_1+1,K_2-1} \\
&\quad +(1-\theta)\beta_1 Q_{K_1+1,K_2} + \beta_2 Q_{K_1,K_2+1} \qquad (3)
\end{aligned}$$

At steady state (i.e.$t \to \infty$) $\frac{dQ_{K_1,K_2}(t)}{dt} = 0$. Here $Q_n(t)$ is no longer a function of t. In (3) now becomes:

$$Q_{K_1,K_2}(\beta_1+\beta_2+\chi) = \chi Q_{K_1-1,K_2} + \theta\beta_1 Q_{K_1+1,K_2-1} + (1-\theta)\beta_1 Q_{K_1+1,K_2} + \beta_2 Q_{K_1,K_2+1} \qquad (4)$$

This equation is known as the steady-state difference equation. Now, the boundary condition for this model is as follows:

When, $K_1 = 0$, (4) becomes $(\chi+\beta_2)Q_{0,K_2} = \theta\beta_1 Q_{1,K_2-1} + (1-\theta)\beta_1 Q_{1,K_2} + \beta_2 Q_{0,K_2+1}$ (5)

When, K_2=0, (4) becomes $(\chi+\beta_1)Q_{K_1,0} = \chi Q_{K_1-1,0} + (1-\theta)\beta_1 Q_{K_1+1,0} + \beta_2 Q_{K_1,1}$ (6)

When, $K_1 = 0, K_2 = 0$, (4) becomes $\lambda Q_{0,0} = (1-\theta)\beta_1 Q_{1,0} + \beta_2 Q_{0,1}$ (7)

3.2. Probability generating function

The probability-generating function serves as a crucial tool for analyzing and understanding the properties of random variables. This function enables researchers to capture the underlying probability distribution of a system. We define the probability-generating function as follows:

$$M_{K_1,K_2}(x_1,x_2) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} {x_1}^{k_1}{x_2}^{k_2}Q_{k_1,k_2};\ M_{K_1,0}(x_1) = \sum_{k_1=0}^{\infty} {x_1}^{k_1}Q_{k_1,0}$$
$$M_{0,K_2}(x_2) = \sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{0,k_2};M_{K_1,1}(x_1) = \sum_{k_1=0}^{\infty} {x_1}^{k_1}Q_{k_1,1}\ ;\ M_{1,K_2}(x_2) = \sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{1,k_2} \quad (8)$$

3.3. Solution of the steady-state probability

Now simplifying the left-hand side of (4) we get:

$$(\beta_1+\beta_2+\chi)\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty} {x_1}^{k_1}{x_2}^{k_2}Q_{k_1,k_2} = (\beta_1+\beta_2+\chi)\sum_{k_1=1}^{\infty} {x_1}^{k_1}\{\sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{k_1,k_2} - Q_{k_1,0}\} \quad (9)$$

$\vartheta = (\beta_1+\beta_2+\chi)$. Hence:

$$\vartheta\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty} {x_1}^{k_1}{x_2}^{k_2}Q_{k_1,k_2} = \vartheta\left[\sum_{k_1=1}^{\infty} {x_1}^{k_1}\sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{k_1,k_2} - \sum_{k_1=1}^{\infty} {x_1}^{k_1}Q_{k_1,0}\right]$$
$$= \vartheta\left[M_{K_1,K_2}(x_1,x_2) - M_{0,K_2}(x_2) - M_{K_1,0}(x_1) + Q_{0,0}\right] \quad (10)$$

The first item on the RHS of (4) is:

$$\vartheta\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty} {x_1}^{k_1}{x_2}^{k_2}Q_{k_1-1,k_2} = \vartheta x_1\left[\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty} {x_1}^{k_1-1}{x_2}^{k_2}Q_{k_1-1,k_2}\right]$$
$$= \vartheta x_1\left[\sum_{k_1=0}^{\infty} {x_1}^{k_1}\sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{k_1,k_2}\right] - \vartheta x_1\sum_{k_1=0}^{\infty} {x_1}^{k_1}Q_{n_1,0}$$
$$= \vartheta x_1 G_{k_1,k_2}(k_1,k_2) - \vartheta x_1 G_{k_1,0}(k_1) \quad (11)$$

The second term on the RHS of (4) is:

$$\theta\beta_1 Q_{k_1+1,k_2-1} = \theta\beta_1\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty} {x_1}^{k_1}{x_2}^{k_2}\,Q_{k_1+1,k_2-1}$$
$$= \theta\beta_1{x_1}^{-1}x_2\left(\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} {x_1}^{k_1}{x_2}^{k_2}Q_{k_1,k_2} - \sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{0,K_2} - x_1\sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{1,K_2}\right)$$
$$= \theta\beta_1{x_1}^{-1}x_2 G_{k_1,k_2}(x_1,x_2) - \theta\beta_1{x_1}^{-1}x_2 G_{0,k_2}(x_2) - \theta\beta_1 x_2 G_{1,k_2}(x_2) \quad (12)$$

The third term on the RHS of (4) is:

$$(1-\theta)\beta_1 Q_{K_1+1,K_2} = (1-\theta)\beta_1\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty} {x_1}^{k_1}\,{x_2}^{k_2}Q_{K_1+1,K_2}$$
$$= (1-\theta)\beta_1 x_1^{-1}\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty} {x_1}^{k_1+1}\,{x_2}^{k_2}Q_{K_1+1,K_2} \quad (13)$$

Let $\gamma = (1-\theta)\beta_1 x_1^{-1}$ then (13) become,

$$(1-\theta)\beta_1 Q_{K_1+1,K_2} = \gamma\sum_{k_1=1}^{\infty} {x_1}^{k_1+1}\left[\sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{K_1+1,K_2} - Q_{K_1+1,0}\right]$$
$$= \gamma\left[G_{K_1,K_2}(x_1,x_2) - G_{0,k_2}(x_2) - x_1 G_{1,K_2}(x_2) - G_{k_1,0}(x_1) + Q_{0,0} + x_1 Q_{1.0}\right] \quad (14)$$

The fourth item on the RHS of (14) is,

$$\beta_2 Q_{k_1,k_2+1} = \beta_2{x_2}^{-1}\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty} {x_1}^{k_1}{x_2}^{k_2+1}Q_{k_1,k_2+1}$$
$$= \beta_2{x_2}^{-1}\left[\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} {x_1}^{k_1}{x_2}^{k_2}Q_{k_1,k_2} - \sum_{k_2=0}^{\infty} {x_2}^{k_2}Q_{0,k_2} - \sum_{k_1=0}^{\infty} {x_1}^{k_1}Q_{k_1,0} + Q_{0,0} - x_2 Q_{0,1}\right]$$

Hence $\beta_2 Q_{k_1,k_2+1}$ equals:

$$=\beta_2{x_2}^{-1}\left[G_{k_1,k_2}(x_1x_2)\right] - G_{0,k_2}(x_2) - G_{k_1,0}(x_1) + Q_{0,0} - x_2 G_{1,k_2}(x_2) + x_1 Q_{0,1} \quad (15)$$

3.4. Expected number of customers in the system

The expected number of people in the system at steady state for a model with two stations can be obtained as follows:

$$E(K_1,K_2) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} k_1, k_2 Q_{k_1}, Q_{k_2} \quad (16)$$

The equation (16) can be written as:

$$E(K_1,K_2) = \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} k_1, k_2\zeta^{k_1}G_{0,k_2}(1)\zeta^{k_2}G_{k_1,0}$$
$$E(K_1,K_2) = (1-\zeta_1)(1-\zeta_2)\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} k_1, k_2\zeta^{k_1}\zeta^{k_2}$$

Where $G_{0,k_2}(1) = 1-\zeta_1 \& G_{k_1,0}(1) = 1-\zeta_2$,

$$E(K_1,K_2) = (1-\zeta_1)(1-\zeta_2)\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} k_1, k_2\zeta^{k_1}\zeta^{k_2} = \frac{\zeta_1\zeta_2}{(1-\zeta_1)(1-\zeta_2)} \quad (17)$$

3.5. Mean waiting time

The mean waiting time in a two-station system can be calculated using a steady-state analysis, providing valuable insights into system performance. This calculation takes into account the arrival rates, service rates, and other key factors that influence waiting times. By determining the mean waiting time, researchers and practitioners can optimize system design and improve overall efficiency.

$$E\,(W)= \frac{(\beta_1 x_1-\beta_1)\frac{Q_0}{x_1}}{(\beta_2+\chi-\chi x_2-\beta_2 x_2^{-1})} = \frac{(\beta_1 x_1-\beta_1)Q_0}{(\beta_2 x_2+\chi x_2-\chi x_2{}^2-\beta_2} \quad (18)$$

4. NUMERICAL ILLUSTRATION

In this section, theoretical results are justified with suitable numerical results. In order to study the effect of arrival rate, service rate at node 1 and service rate at node 2. From Figure 2 and Table 1, we can observe that the arrival rate increases when AI is driven in this model then mean waiting customers speedily decreases.

Table 1. Arrival rate vs. mean waiting time
$\beta_1 = 0.5, \beta_2 = 0.4, \chi = 0.2, \theta = 0.8$

X	Server1	Server2	System
1	1.7884	2.4564	4.2448
2	0.65	2.2653	2.9153
3	0.9907	2.006	2.9967
4	0.6505	1.856	2.5065
5	0.4808	1.75	2.2308
6	0.2056	1.6503	1.8559

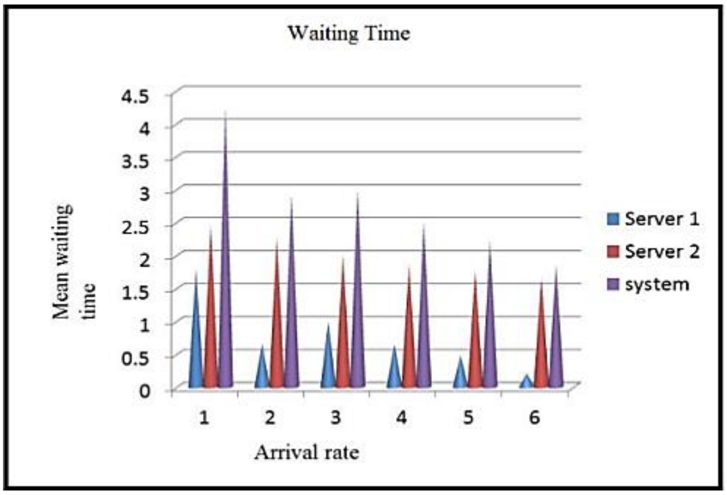

Figure 2. AI-driven, arrival rate vs. waiting time

5. CONCLUSION

This paper introduces an innovative approach to tandem queue analysis using AI techniques. AI-driven tandem queue analysis represents a transformative force across industries, offering multifaceted advantages. In this article, we consider a tandem queue with two service stations using AI techniques. The steady-state difference equation for this model is derived and a mean number of customers are derived. Companies can revolutionize operations through automated queueing, streamlining data analysis, accumulation, and AI-driven customer solutions to drive business success. Existing models may lack robustness to such uncertainty, leading to suboptimal performance in practice. Recent observations suggest that the efficiency of tandem queues can significantly fluctuate due to unpredictable customer reneging behavior. Our findings provide conclusive evidence that this phenomenon is associated with dynamic changes in service conditions and arrival rates, not due to evaluated numbers of servers or static queue parameters. Future studies may explore the integration of robust machine learning techniques with feasible ways of producing adaptive, real-time optimizations to further enhance queue efficiency and customer satisfaction. Future research should explore techniques for robust queue management that can handle uncertainty effectively.

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