Novel similarity measures for Fermatean fuzzy sets and its applications in image processing

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ABSTRACT

Digital imaging is growing in our day-to-day life ranging from selfies to medical imaging. The extended applications of the field open doors for the researchers in the present-day context. The extraction of useful information from digital images is crucial because it depends on the various characteristics of the image. Fuzzy theory provides a better understanding of the image characteristics and, thus extracts meaningful information, even under uncertain situations. The present study reports the Fermatean fuzzy sets (FFSs) application in image processing while proposing similarity measures. These similarity measures highlight the perfect and precise results from an image while using multiple parameters of the image for information extraction. The study concludes that the proposed similarity measures provide a better estimation of data from an image used in image processing problems.

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1. INTRODUCTION

In earlier days, data was reported in the words and the various analysis were done to extract the information from the data collected. Storage and handling of the data in that form was very complex and, in many cases, the collection of data was very lengthy. In [1], [2], for the cut shorten, now-a-days, data is captured through the imaging processes. An image is referred to as the object having the information about the product, process or about an environment situation. A fine quality image has high contrast and brightness while a poor-quality image has low contrast and poorly defined boundaries between the edges. The image processing considers the image acquisition, transmission, and extraction of useful information. In the present context, the adoption of innovative tools in the image sensing field becomes of great importance. It contributes through several ways i.e., capturing the moment without losing the necessary data, lowering the transformational time estimation time, and being less impacted by environmental conditions. In [3], [4], the involvement of computational as well mathematical algorithms makes the job easier of information extraction from any image. Further, these tools also support the decision maker in continuous monitoring of the object (if necessary) and understanding of the trends.

The application of fuzzy sets in the estimation of information from an image is the most common type techniques for image processing. The fuzzy interventions enable the decision makers to explore the image characteristics and estimate the information. According to Nguyen [5], fuzzy set involvement helps in estimating the similarity and the algorithms provides the promising results. Still, there are affects in processing the information from an image especially in medical purpose imaging and CCTV footages.

According to Garg and Singh [6], both the imaging processes are full of uncertainty and the images are of noisy and blurry type. The present study proposed the application of Fermatean fuzzy sets (FFSs) in estimation of images. The reason for using the FFS is that these sets handled uncertainty very well than other generalized fuzzy sets. According to Senapati and Yager [7], the FFS applications are most commonly used for reducing the impact of uncertainty in case of blurred images. They compared FFS with Pythagorean fuzzy set (PFS) and intuitionistic fuzzy sets (IFSs), and investigated that FFS handled uncertain situations very well than other generalized measures. The proposed measures are competent enough to acknowledge the right image with higher level of uncertainty and efficiently process the noisy images. The images pixels are categorized into three factors such as the membership degree, the non-membership degree, and the degree of hesitancy.

2. LITERATURE REVIEW

Weken *et al.* [8], in the real world, day-to day technological advancements have both positive as well as negative impacts on information management process. While discussing the positive impacts, it supports the decision-making process and makes responsive to all concerns. Whereas, the negative impacts consider the implications in the acquisition, transmission, and extraction process in information management. Lakshmiprabha [9], image sensing is one of the important areas and presently contributes in making the effective decisions in real life situations. In the image sensing, the images are captured for getting the initial data and then the data transmission tools and information extraction methods are applied for getting the useful information. The literature on image sensing depicts that the information quality depends on parameters such as characteristics of the image capturing device, the environmental condition and the applied extraction method accuracy.

In the image processing, there is no set rule to identify the right image in different conditions of perturbations and irregularities. For extracting the perfect and precise information, the fuzzy sets are first used by Zadeh [10] in 1965. In [11], [12], the application of fuzzy set addresses the problem of uncertain information extracted from an image. Later on, the IFSs was introduced as an generalization of fuzzy sets by Atanassov [13] in 1986. In [14], [15], the IFS are having more potential than fuzzy set to deal with uncertain and incomplete information and has applications in image processing. The PFSs deployment in the field of image sensing explores the more precise information extraction through reducing/eliminating the chances of uncertainties in many real-life situations have been discussed in [16]–[20]. In the fuzzy set's algorithms, similarity measure is estimated. The similarity measure provides the quantitative estimation of two images or two patches present in an image. In general, the similarity measure is a technique used to evaluate the degree of similarity between two fuzzy sets. These measures are commonly estimated to get the following data such as: information retravel from an image, classification of images, detecting changes (if any), and evaluation of image quality.

According to Lee-Kwang *et al.* [21], similarity measures provide an efficient tool to analyze the degree of closeness between two sets of objects. In [22], [23], the set of input images captures from different environmental conditions and angles. But the degree of similarity between the images is important with an aim to consider whether images belong to the same category. Sharma and Tripathi [24], generally, one image is the reference image to other target images and is compared. The objective is to compare target images with the reference image to understand the degree of similarity. The growing demand of efficient algorithms especially in image processing attracts the attention of researchers. Hussain [25], computational image processing techniques are very popular to better recognize the image. This can be done by the help of efficient similarity measures that accurately identify the images with uncertainty.

The literature depicts that the similarity measures have great importance in the field of image processing for the purpose of evaluation and comparison of various algorithms designed to solve particular problems. The comparison between the two images is limited to the particular image regions of each image. Such situations usually occurred in medical image and computer vision problems in which identification of right image is must for target image with respect to the reference image. According to Agheli *et al.* [26], image similarity measures play an important role in the identification of duplicate product detection, visual search, and recommendation tasks. These measures easily quantify the similarity pair of images and return a value that tell how visually similar images are?

3. PROPOSED METHOD

In real world, lot of fuzzy and uncertain information is available. Before the introduction of fuzzy sets and its generalizations, information context dealt with only crisp numbers. As an advancement, the FFSs are introduced to handle uncertain information more easily than Pythagorean and IFSs. For the membership values such as (0.9, 0.6), both PFSs as well as IFSs does not follow the constraint condition. According to

Senapati and Yager [27], the constraint condition is being followed by the FFSs in higher levels of uncertainties i.e., the membership value of 0.9 and 0.6 reveals the positive and negative membership degrees of an image respectively. The application of FFS has been used by the researchers [28]–[31], across the domain for the purpose of decision making. The FSS is referred as the generalization of intuitionistic fuzzy set and estimated as:

$$Im\,a\,ge(Im) = \{< x, \mu Im_{Im}$$

Where, μ_{Im} where ν_{Im} are the degree of membership and non-membership such that $0 \leq {\mu_{Im}}^3 + {\nu_{Im}}^3 \leq 1$ with ${\pi_{Im}}^3 = 1 - ({\mu_{Im}}^3 + {\nu_{Im}}^3)$ is the hesitancy/uncertainty of FSS Im.

3.1. Proposed similarity measures based on Pythagorean fuzzy set

Four different similarity measures based on FFS along with the axioms have been proposed as: Let $Im\,a\,ge(Im\_1) = \{< x_i, \mu_{Im\_1}(x_i), \nu_{Im\_1}(x_i) > \mid x_i \in X\}$ and $Im\,a\,ge(Im\_2) = \{< x_i, \mu_{Im\_2}(x_i), \nu_{Im\_2}(x_i) > \mid x_i \in X\}$ be the two Fermatean fuzzy images sets in the universe set of images/discourse $X = \{x_1, x_2, \ldots, x_n\}$. The proposed cosine similarity measures are given as"

$$Sim\_1(Im\_1, Im\_2) = \frac{1}{2n}\sum_{i=1}^{n}\begin{pmatrix} cos\frac{\pi}{2}\left|{\mu_{Im\_1}}^3(x_i) - {\mu_{Im\_2}}^3(x_i)\right| + \\ cos\frac{\pi}{2}\left|{\nu_{Im\_1}}^3(x_i) - {\nu_{Im\_2}}^3(x_i)\right| \end{pmatrix} \text{without hesitancy} \quad (1)$$

$$Sim\_2(Im\_1, Im\_2) = \frac{1}{3n}\sum_{i=1}^{n}\begin{pmatrix} cos\frac{\pi}{2}\left|{\mu_{Im\_1}}^3(x_i) - {\mu_{Im\_2}}^3(x_i)\right| + \\ cos\frac{\pi}{2}\left|{\nu_{Im\_1}}^3(x_i) - {\nu_{Im\_2}}^3(x_i)\right| + \\ cos\frac{\pi}{2}\left|{\pi_{Im\_1}}^3(x_i) - {\pi_{Im\_2}}^3(x_i)\right| \end{pmatrix} \text{with hesitancy} \quad (2)$$

In many real-life applications, measures have assign weights and are defined as:

$$Sim\_3(Im\_1, Im\_2) = \frac{1}{2n}\sum_{i=1}^{n} w_i\begin{pmatrix} cos\frac{\pi}{2}\left|{\mu_{Im\_1}}^3(x_i) - {\mu_{Im\_2}}^3(x_i)\right| + \\ cos\frac{\pi}{2}\left|{\nu_{Im\_1}}^3(x_i) - {\nu_{Im\_2}}^3(x_i)\right| \end{pmatrix} \text{Without hesitancy} \quad (3)$$

$$Sim\_4(Im\_1, Im\_2) = \frac{1}{3n}\sum_{i=1}^{n} w_i\begin{pmatrix} cos\frac{\pi}{2}\left|{\mu_{Im\_1}}^3(x_i) - {\mu_{Im\_2}}^3(x_i)\right| + \\ cos\frac{\pi}{2}\left|{\nu_{Im\_1}}^3(x_i) - {\nu_{Im\_2}}^3(x_i)\right| + \\ cos\frac{\pi}{2}\left|{\pi_{Im\_1}}^3(x_i) - {\pi_{Im\_2}}^3(x_i)\right| \end{pmatrix} \text{With hesitancy} \quad (4)$$

These candidate similarity measures (1)-(4) must satisfy the following axioms as:

a) $0 \leq Sim\_1(Im\_1, Im\_2) \leq 1$
b) $Sim\_1(Im\_1, Im\_2) = 1 \Leftrightarrow [Im\_1 = Im\_2]$
c) $Sim\_1(Im\_1, Im\_2) = Sim\_1(Im\_2, Im\_1)$
d) If $Im\,a\,ge(Im\_3)$ is a FFS in X and $[Im\_1 \subseteq Im\_2 \subseteq Im\_3)$, then

$$Sim\_1(Im\_1, Im\_3) \leq Sim\_1(Im\_1, Im\_2) \text{ and } Sim\_1(Im\_1, Im\_3) \leq Sim\_1(Im\_2, Im\_3)$$

The proofs of these axioms for the proposed measures (1)-(4) have been presented as:
Proof 1.
Since, $0 \leq cos\,\theta \leq 1$

$$\text{Thus, } 0 \leq \begin{pmatrix} cos\frac{\pi}{2}\left|{\mu_{Im\_1}}^3(x) - {\mu_{Im\_2}}^3(x)\right| + \\ cos\frac{\pi}{2}\left|{\nu_{Im\_1}}^3(x) - {\nu_{Im\_2}}^3(x)\right| \end{pmatrix} \leq 2$$

$$\Rightarrow\ 0 \leq \frac{1}{2}\begin{pmatrix} cos\frac{\pi}{2}\left|{\mu_{Im\_1}}^3(x) - {\mu_{Im\_2}}^3(x)\right| + \\ cos\frac{\pi}{2}\left|{\nu_{Im\_1}}^3(x) - {\nu_{Im\_2}}^3(x)\right| \end{pmatrix} \leq 1$$

$$\Rightarrow\ 0 \leq \frac{1}{2n}\begin{pmatrix} cos\frac{\pi}{2}\left|{\mu_{Im\_1}}^3(x) - {\mu_{Im\_2}}^3(x)\right| + \\ cos\frac{\pi}{2}\left|{\nu_{Im\_1}}^3(x) - {\nu_{Im\_2}}^3(x)\right| \end{pmatrix} \leq 1$$

Therefore, $0 \leq Sim\_1(Im\_1, Im\_2) \leq 1$

Proof 2.

$$\Leftrightarrow \quad \frac{1}{2n}\sum_{i=1}^{n}\begin{pmatrix} cos\frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_2}{}^3(x_i)\right| + \\ cos\frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_2}{}^3(x_i)\right| \end{pmatrix} = 1$$

$$\Leftrightarrow \quad \begin{pmatrix} cos\frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_2}{}^3(x_i)\right| + \\ cos\frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_2}{}^3(x_i)\right| \end{pmatrix} = 2$$

$$\Leftrightarrow \quad \left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_2}{}^3(x_i)\right| = 0 \text{ and } \left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_2}{}^3(x_i)\right| = 0$$

$$\Leftrightarrow \quad \mu_{Im\_1}(x_i) = \mu_{Im\_2}(x_i) \text{ and } \nu_{Im\_1}(x_i) = \nu_{Im\_2}(x_i)$$

$$\Leftrightarrow \quad Im\_1 = Im\_2$$

Proof 3.

Since cosine function is symmetrical, proof is obvious.

Proof 4.

Given that $Im\,a\,ge(Im\_3)$ is a PFS in X and $[Im\_1 \subseteq Im\_2 \subseteq Im\_3)$; $\forall\ x_i \in X$. We have, $0 \le \mu_{Im\_1}(x_i) \le \mu_{Im\_2}(x_i) \le \mu_{Im\_3}(x_i) \le 1$ and $1 \ge \nu_{Im\_1}(x_i) \ge \nu_{Im\_2}(x_i) \ge \nu_{Im\_3}(x_i) \ge 0$

$$\Rightarrow 0 \le \mu_{Im\_1}{}^3(x_i) \le \mu_{Im\_2}{}^3(x_i) \le \mu_{Im\_3}{}^3(x_i) \le 1 \text{ and } 1 \ge \nu_{Im\_1}{}^3(x_i) \ge \nu_{Im\_2}{}^3(x_i) \ge \nu_{Im\_3}{}^3(x_i) \ge 0$$

$$\Rightarrow \left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_2}{}^3(x_i)\right| \le \left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_3}{}^3\right|$$
$$\left|\mu_{Im\_2}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right| \le \left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right|$$

And:

$$\Rightarrow \left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_2}{}^3(x_i)\right| \le \left|\nu_{Im\ \ 1}{}^3(x_i)-\nu_{Im\ \ 3}{}^3(x_i)\right|;$$
$$\left|\nu_{Im\_2}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right| \le \left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right|$$

$$\Rightarrow \frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_2}{}^3(x_i)\right| \le \frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right|$$
$$\frac{\pi}{2}\left|\mu_{Im\_2}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right| \le \frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right|$$

And:

$$\Rightarrow \frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_2}{}^3(x_i)\right| \le \frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right|;$$
$$\frac{\pi}{2}\left|\nu_{Im\_2}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right| \le \frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right|$$

$$\Rightarrow cos\left(\frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_2}{}^3(x_i)\right|\right) \le cos\left(\frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right|\right)$$
$$cos\left(\frac{\pi}{2}\left|\mu_{Im\_2}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right|\right) \le cos\left(\frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right|\right)$$

And:

$$\Rightarrow cos\left(\frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_2}{}^3(x_i)\right|\right) \le cos\left(\frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right|\right);$$
$$cos\left(\frac{\pi}{2}\left|\nu_{Im\_2}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right|\right) \le cos\left(\frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right|\right)$$

Adding the above equations:

$$\left[cos\left(\frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_2}{}^3(x_i)\right|\right) + cos\left(\frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_2}{}^3(x_i)\right|\right)\right] \le$$
$$\left[cos\left(\frac{\pi}{2}\left|\mu_{Im\_1}{}^3(x_i)-\mu_{Im\_3}{}^3(x_i)\right|\right) + cos\left(\frac{\pi}{2}\left|\nu_{Im\_1}{}^3(x_i)-\nu_{Im\_3}{}^3(x_i)\right|\right)\right]$$

$$\frac{1}{2n}\sum_{i=1}^{n}\left[cos\left(\frac{\pi}{2}\left|\mu_{Im\_1}{}^{3}(x_i)-\mu_{Im\_2}{}^{3}(x_i)\right|\right)\ +\ cos\left(\frac{\pi}{2}\left|\nu_{Im\_1}{}^{3}(x_i)-\nu_{Im\_2}{}^{3}(x_i)\right|\right)\right]\le$$
$$\frac{1}{2n}\sum_{i=1}^{n}\left[cos\left(\frac{\pi}{2}\ \left|\mu_{Im\_1}{}^{3}(x_i)-\mu_{Im\_3}{}^{3}(x_i)\right|\right)\ +\ cos\left(\frac{\pi}{2}\left|\nu_{Im\_1}{}^{3}(x_i)-\nu_{Im\_3}{}^{3}(x_i)\right|\right)\right]$$

$$\Rightarrow Sim\_1(Im\_1\,,Im\_3)\le Sim\_1(Im\_1\,,Im\_2)$$

Similarly, $Sim\_1(Im\_1\,,Im\_3)\le Sim\_1(Im\_2\,,Im\_3)$. From the results, it has been proved that the candidate similarity measures are the valid measures and are suitable for the decision making.

4. NUMERICAL VALIDATION OF THE PROPOSED MEASURES

Consider that $Im\,a\,ge(Im\_1)$, $Im\,a\,ge(Im\_2)$, and $Im\,a\,ge(Im\_3)$ be the Fermatean fuzzy image sets in the universe set of images/discourse $X=\{x_1,x_2,\ldots.,x_n\}$. Let us consider that:

$$Im\,a\,ge(Im\_1)=\ \{\langle x_1,\ 0.9,\ 0.6\rangle,\ \langle x_2,\ 0.8,\ 0.6\rangle,\langle x_3,\ 0.95,\ 0.5\rangle\ \}$$

$$Im\,a\,ge(Im\_2)=\ \{\langle x_1,\ 0.8,\ 0.7\rangle,\ \langle x_2,\ 0.85,\ 0.7\rangle,\langle x_3,\ 0.85,\ 0.65\rangle\ \}$$

$$Im\,a\,ge(Im\_3)=\ \{\langle x_1,\ 0.9,\ 0.6\rangle,\ \langle x_2,\ 0.75,\ 0.8\rangle,\langle x_3,\ 0.8,\ 0.7\rangle\ \}$$

On the basis of this information, the numerical values of the proposed similarity measures are given as in Table 1. Table 1 reports that Im_2 and Im_3 have higher chances of similarity for the applicability of all the proposed similarity measures.

Table 1. Similarity measures

Similarity measures	$(Im\_1\,,Im\_2)$	$(Im\_2\,,Im\_3)$	$(Im\_1\,,Im\_3)$
Sim_1	0.965061	0.970627	0.947035
Sim_2	0.967071	0.979043	0.956544
Sim_3	0.321933	0.322253	0.320804
Sim_4	0.322739	0.325322	0.322766

4.1. Applications of the proposed measures in the image processing

Let $Im\,a\,ge(Im\_1)$, $Im\,a\,ge(Im\_2)$, and $Im\,a\,ge(Im\_3)$ be the images described in FFS. Also, $X=\{x_1,x_2,x_3\}$ be universal set of images defined in FFS. Taking:

$$Im\,a\,ge(Im\_1)=\ \{\langle x_1,\ 0.8, 0.7\rangle,\ \langle x_2,\ 0.8,\ 0.6\rangle,\langle x_3,\ 0.95,\ 0.5\rangle\ \}$$

$$Im\,a\,ge(Im\quad 2)=\ \{\langle x_1,\ 0.8, 0.7\rangle,\ \langle x_2,\ 0.85,\ 0.7\rangle,\langle x_3,\ 0.85,\ 0.65\rangle\ \}$$

$$Im\,a\,ge(Im\_3)=\ \{\langle x_1,\ 0.9, 0.6\rangle,\ \langle x_2,\ 0.75,\ 0.8\rangle,\langle x_3,\ 0.8,\ 0.7\rangle\ \}$$

$$X=\ \{\langle x_1,\ 0.95, 0.5\rangle,\ \langle x_2,\ 0.8,\ 0.75\rangle,\langle x_3,\ 0.7,\ 0.7\rangle\ \}$$

Also, taking weights as: 0.5, 0.3, and 0.2 respectively. The calculation is given in Table 2.

Table 2. Estimation of similarity measure using proposed measures

Similarity measures	$(Im\_1\,,X)$	$(Im\_2\,,X)$	$(Im\_3\,,X)$
Sim_1	0.854748	0.94713	0.985751
Sim_2	0.91085	0.956734	0.985738
Sim_3	0.301703	0.312336	0.328627
Sim_4	0.306934	0.306934	0.329309

5. RESULTS AND DISCUSSION

This work proposed new similarity measures based on FFSs. The proposed measures satisfy the axioms (1-4) to become candidate measures, which showed that the proposed measures are valid, given in the proofs of theorems. Further, the reliability of the proposed measures has been acknowledged through numerical computations with applications in the given image processing problem. From Figure 1, it is observed that the Im_3 resembles to be the best suited image with respect to the reference image. The similarity measures can be easily applicable on the problems of image processing where there is uncertainty in the acknowledgement of images.

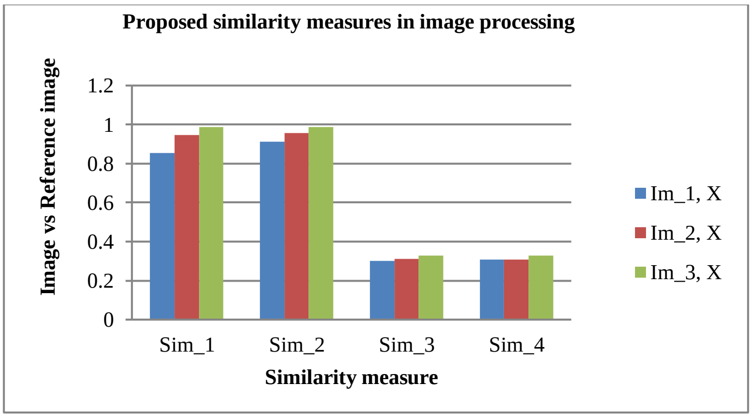

Figure 1. The proposed similarity measures with respect to the given images

6. CONCLUSION

The present study explores the critical facets of images processing using the Fermatean fuzzy-based image sensing system. Four similarity measures have been proposed that highlights the applications of image processing in getting the perfect and precision information while using multiple uncertain parameters of an image. From the results, it is concluded that the proposed similarity measures have the potential to deal with the problems of uncertainty with higher degree. The future prospective of the proposed work can be utilized to design a recommender system by considering for image features for better extraction of the information under uncertain condition.

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