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Determining student scaffolding levels in geometry problem-solving: a fuzzy inference system using Mamdani method

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ABSTRACT

This study aims to adopt a fuzzy logic inference system using Mamdani method to determine the appropriate scaffolding level for students based on errors in solving geometry problems. The fuzzy inference system (FIS) assessed the students' understanding and provided scaffolding to address specific learning needs by analyzing common mistakes in geometry problem-solving. This method optimized the educational process by offering personalized support, enhancing students' problem-solving skills, and reducing error rates. The data processing criteria using the FIS involved analyzing the scores of mathematics education students at a private university in Malang when solving geometry problems, based on Pólya's stages. Mamdani was used to provide recommendations based on students' cognitive data while solving geometry problems, in line with scaffolding components. The results showed that the system personalized scaffolding levels for students working on geometry problems. This contributed to the field of educational technology by presenting a new approach to adaptive learning and emphasized the importance of personalized pedagogical support. The approach was particularly beneficial for geometry teachers who provided individualized scaffolding based on students' errors, contributing to improved learning outcomes.

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1. INTRODUCTION

Geometry is a core area of mathematics that plays a crucial role in developing students' mathematical thinking. Despite the undeniable significance, this concept remains a challenging topic and is often perceived as difficult by many students at different levels of education [1]. The difficulties are not only limited to understanding basic concepts, but extend to the application of geometry in problem solving and spatial reasoning. Recent studies suggest that challenges in learning geometry can result from various factors, including conceptual complexity, the need for strong visualization, and the relationship between geometric and algebraic representations [2], [3].

A variety of innovative strategies and approaches have been developed to address the challenges of learning geometry. An important strategy proven effective is the use of scaffolding, which provides structured support to students in building geometric understanding [4], [5]. Scaffolding in geometry learning can take various forms, spanning from verbal instructions to digital visualization tools. However, the determination of the appropriate level and type of scaffolding remains a challenge, given the diversity of students' abilities, learning styles, and individual needs [6]–[8]. A more systematic and adaptive approach in determining the level is needed to optimize the geometry learning process.

The development of digital technology has opened up new opportunities in geometry learning and the application of more adaptive scaffolding. The fuzzy inference system (FIS) using the Mamdani method provides a promising way to handle the complexity of finding the optimal level in geometry learning. FIS handles uncertainty and ambiguity in the assessment of student error rates and produces more flexible and adaptive decisions than conventional rule-based systems [9], [10]. The application in mathematics education has shown positive results in a variety of contexts, including assessment of student performance, adaptation of learning materials, and personalization of the learning experience [11].

This study aims to develop a FIS using the Mamdani method to determine the appropriate scaffolding level for students, based on their errors in solving geometry problems. The system is expected to assist teachers in providing more targeted and effective support to students to improve understanding and performance in learning geometry [1], [12]. The gap between theory and practice is bridged by integrating knowledge about types of errors [13], [14] and principles of effective scaffolding [6], [15].

In the context of geometry learning, FIS is used to analyze the different types of errors students make and recommend appropriate levels of scaffolding. Recent studies have identified several major categories of errors in geometry, including conceptual, procedural, and visual-spatial errors [14]–[18]. In considering the error characteristics, FIS systems are designed to provide more targeted and effective scaffolding recommendations, helping students overcome specific barriers in geometry understanding.

The development of FIS-based scaffolding systems for geometry learning requires careful integration of learning theory, geometry domain knowledge, and intelligent system design principles. This interdisciplinary approach includes an in-depth analysis of students' cognitive processes in geometry learning, an understanding of the structure and relationships of geometric concepts, and consideration of pedagogical aspects in the application of scaffolding [19]–[22]. Furthermore, the design should also consider the practical aspects of implementation in actual classroom environments, including ease of use for teachers and students, as well as the ability to integrate with existing learning systems.

Evaluating the effectiveness of FIS-based scaffolding systems in learning geometry is a crucial aspect that requires a rigorous methodological approach. Recent studies have shown the importance of using mixed methods that combine quantitative and qualitative analysis to understand the impact of mathematics learning interventions [5], [21]. In the context of scaffolding systems for geometry, evaluation should include improvements in student performance, changes in attitudes toward geometry, development of metacognitive skills, and transfer of understanding to other mathematical domains.

The implementation of FIS-based scaffolding systems must consider the social and cultural context of learning. Recent studies emphasize the importance of culturally responsive approaches in mathematics education, including geometry [23]–[25]. The scaffolding system should be sufficiently flexible to accommodate various perspectives and experiences of students as well as support the development of a positive mathematical identity.

The analysis of errors in geometry problem solving can be enhanced by applying Pólya's problem solving framework, which includes four stages: understanding the problem, planning the solution, executing the plan, and checking back [26]. Recent studies show that integrating Pólya's stages with error analysis can provide deeper insights into difficulties in geometry [27]–[29]. At the problem understanding stage, students often have difficulty interpreting the given geometric information and identifying key elements of the problem [30]. In the planning the solution stage, errors often occur when there is failure to connect relevant geometric concepts or choose inappropriate strategies [31]. During the execution of the plan, procedural and calculation errors often appear, specifically when students apply geometry formulas or theorems [32]. At the recheck stage, many students fail to perform adequate verification or are unable to identify errors in solutions.

The integration of Polya stage-based error analysis into FIS scaffolding systems increases the precision and effectiveness of support. The system provides more targeted and contextualized scaffolding by identifying at which stage students are having difficulty [6]. For example, when a student has difficulty at the problem understanding stage, the system may provide scaffolding that focuses on visual representation or problem decomposition. In contrast, when difficulties arise at the stage of executing the plan, scaffolding can be in the form of step-by-step guidance or reminders about key concepts [9], [10].

This approach helps address specific difficulties in geometry problem solving and promotes students' awareness of cognitive processes during problem solving. Students become more conscious of the strategies and errors by promoting reflection at each step of Pólya's problem-solving framework [5], [21]. Therefore, the learning process focuses on obtaining correct answers and emphasizes the development of transferable metacognitive skills.

The development of FIS-based scaffolding systems for geometry learning offers a promising opportunity to address long-standing challenges in this field. The systems can enhance the learning experience and improve student outcomes by incorporating a thorough understanding of student challenges, the principles of effective scaffolding, and the adaptive capabilities of FIS. However, the realization requires strong interdisciplinary collaboration between mathematics teachers, geometry domain experts, learning system designers, and educational experts. Learning tools addressing difficulties in geometry and enhancing deeper appreciation of mathematics among students are developed through this collaborative effort.

2. METHOD

FIS was created by adapting the rapid application development (RAD) methodology, which includes four stages: prototype design, requirements planning, and iterative construction [33]. According to Pourjavad and Mayorga [34], the fuzzy Mamdani method for calculating control outputs consists of three primary steps. These steps are evaluation, fuzzification, and defuzzification.

2.1. Planning phase

The planning phase starts with the initial stage of the Mamdani method, called fuzzification, where input and output variables are identified and their respective fuzzy sets are defined. In the Mamdani method, both input and output variables are segmented into one or more fuzzy sets. Secondly, rule evaluation incorporates the implication function through min and rule composition. During this stage, the implication function employed is min, and the rule composition results from the inference process. The max method is used among the three inference methods available. The defuzzification stage employs the centroid of area (COA) method for defuzzification within Mamdani's fuzzy rule composition. This process is illustrated by (1).

$$z = \frac{\sum \mu D}{\sum \mu} \tag{1}$$

Where μ represents the membership degree of the fuzzy set and D denotes a precise number.

FIS adopts multi-input single-output (MISO) structure, in which the inputs consist of scaffolding components required to address errors in solving geometry problems, while the output represents the appropriate level. This study refers to Pólya's stages of problem solving, which include understanding the problem, as well as planning, implementing, and re-examining the solution. This study classifies scaffolding components into three categories adjusted to Pólya's stages, namely environmental provisions, explanation, review, restructuring, and developing conceptual thinking, by referring to the [35] scaffolding model. Each component is rated with a certain scale, and the fuzzy system produces output in the form of scaffolding levels divided into initial, advanced, independent, and high independence. This study aims to personalize learning support based on students' errors in solving geometry problems to improve understanding and independence. Anghileri’s [35] scaffolding framework is mapped onto each stage of problem solving to clarify the relationship between scaffolding components and Pólya’s problem-solving stages. The mapping reports how different scaffolding strategies support students at specific cognitive stages. Table 1 provides a detailed description of the scaffolding components.

Table 1. Description of scaffolding components associated with Pólya's problem-solving stages

Anghileri's scaffolding component	Stages of problem solving Polya
Environmental provisions (ep)	Understand the problem: – Visualize the problem – Reorganize facts and problems using own language
Explaining, reviewing, and restructuring (er)	Make a plan: – Make connections from what is known (facts) to the problem – Develop a problem-solving plan or strategy based on concepts related to the problem Carry out the plan: – Apply the concepts and strategies that have been chosen to solve the problem. – Perform calculations carefully and thoroughly.
Developing conceptual thinking (dc)	Look back: – Answer problems based on problem-solving and reflect on the answers that have been made. – Look for other alternatives to solve the problem or the possibility of other concepts related to the problem at hand.

2.2. Prototype design

During the prototype design phase, each criterion was evaluated and classified into three categories: strong, moderate, and weak, each with its own value range. The domain interval of each input variable is adjusted to the number of descriptors from the variable. A descriptor was assessed at a maximum of

10 points. The environmental provisions variable has 2 descriptors since the domain interval is [0,20]. The explaining, reviewing, and restructuring variable has 4 descriptors, and the domain interval is [0,40]. The developing conceptual thinking variable has 2 descriptors, and the domain interval is [0,20]. Table 2 displays the range of values for the three criteria. The range serves as data to assess the degree of student scaffolding.

2.3. Iterative construction

During the prototype development stage, the main component display of the Mamdani FIS was designed by focusing on four functional modules, as shown in Figure 1. The overall structure of the proposed Mamdani FIS is illustrated in Figure 2. The architecture progresses from left to right, transitioning from three inputs (ep, er, and dc) to the output (scaffolding level).

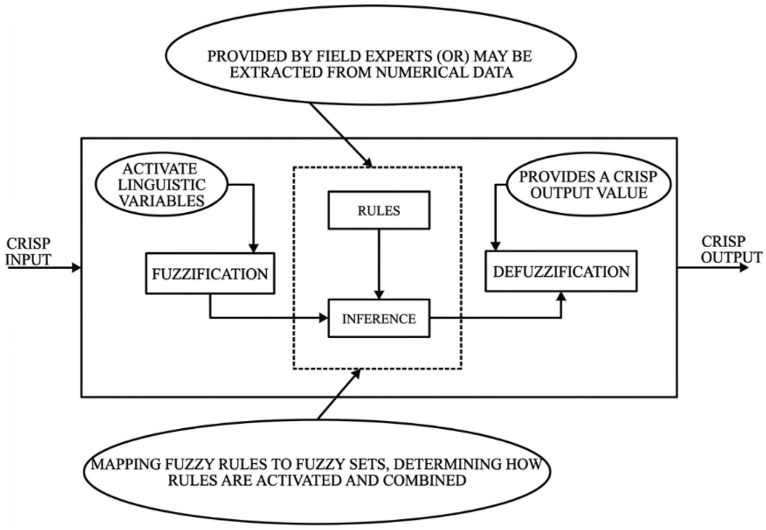

Figure 1. Diagram of Mamdani FIS model used in this study

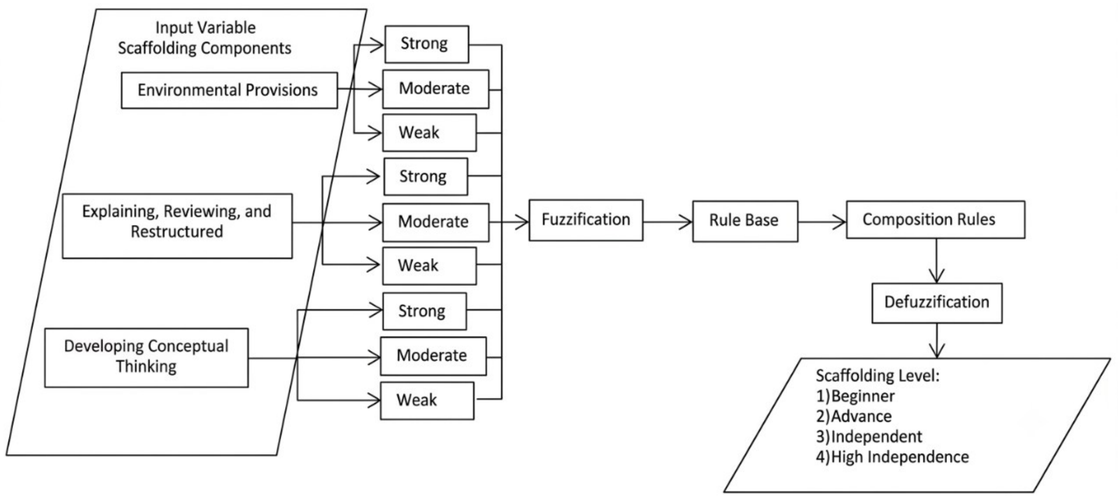

Figure 2. Diagram the inputs and outputs of the proposed Mamdani FIS model used to identify students' scaffolding levels

3. RESULTS AND DISCUSSION

The fuzzy system was created using the Mamdani method by combining the primary functional modules, namely fuzzification, inference, rule evaluation, and defuzzification. Each module plays a crucial role in processing student performance data and converting the context into meaningful scaffolding-level recommendations. This structured approach allows the system to manage uncertainties in student errors and offer adaptive support aligned with instructional requirements.

3.1. Fuzzification

Fuzzy logic is utilized to analyze real-time experimental inputs and assess their membership levels [36]. Fuzzification is a component of fuzzy logic that transforms crisp data into fuzzy values ranging from 0 to 1 [37]. In this context, a value of 1 indicates full (100%) membership in the fuzzy set, while a value of 0 indicates no (0%) membership. Table 2 presents details on linguistic variables linked to each parameter, facilitating a thorough understanding of the fuzzy sets. Fuzzy inputs should be utilized with Mamdani FIS. To transform crisp inputs into fuzzy ones, membership functions are employed to define input sets. Table 2 illustrates various forms of membership functions that represent different levels of fuzziness across various scenarios.

Table 2. Fuzzy linguistic variables for each parameter

Parameter	Code	Linguistic variable	Range	Representation
Input (3)	ep	Strong (T)	12 – 20	Figure 3
		Moderate (S)	04 – 16	
		Weak (R)	00 – 08	
	er	Strong (T)	24 – 40	Figure 4
		Moderate (S)	08 – 32	
		Weak (R)	00 – 16	
	dc	Strong (T)	12 – 20	Figure 5
		Moderate (S)	04 – 16	
		Weak (R)	00 – 08	
Output	Level	Beginner level (LA)	00 – 30	Figure 6
		Advanced level (LL)	10 – 50	
		Independent level (LI)	30 – 70	
		High independence (LKT)	50 – 80	

3.1.1. Input variable environmental provisions: developing conceptual thinking

The range for each category is calculated based on the analysis of student test results when working on geometry problems and adjusted according to Table 1. Figures 3 to 6 display fuzzy membership functions created with Microsoft Excel. The four figures illustrate a membership function composed of a combination of trapezoidal shapes. This is commonly used because of its simplicity and can be represented using a straight line [38]. To enhance reproducibility, the exact parameter settings of the trapezoidal membership functions are explicitly provided in Table 3. These parameters define the lower, upper, and overlapping regions for each fuzzy set.

Table 3. Parameters of the trapezoidal membership function

Variable	Fuzzy set	a	b	c	d
ep	Weak	0	0	4	8
ep	Moderate	4	8	12	16
ep	Strong	12	16	20	20
er	Weak	0	0	8	16
er	Moderate	8	16	24	32
er	Strong	24	32	40	40
dc	Weak	0	0	4	8
dc	Moderate	4	8	12	16
dc	Strong	12	16	20	20

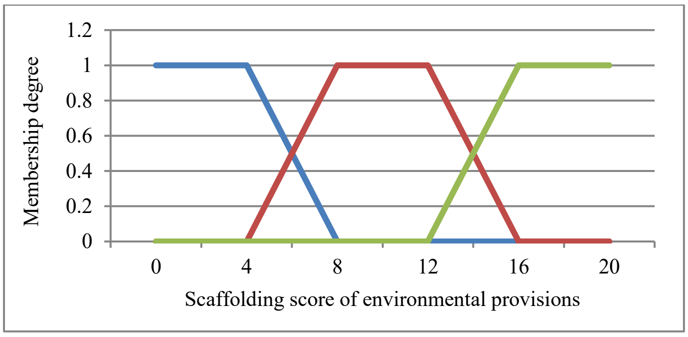

Figure 3. Model performance using a trapezoidal membership function for ep input variable

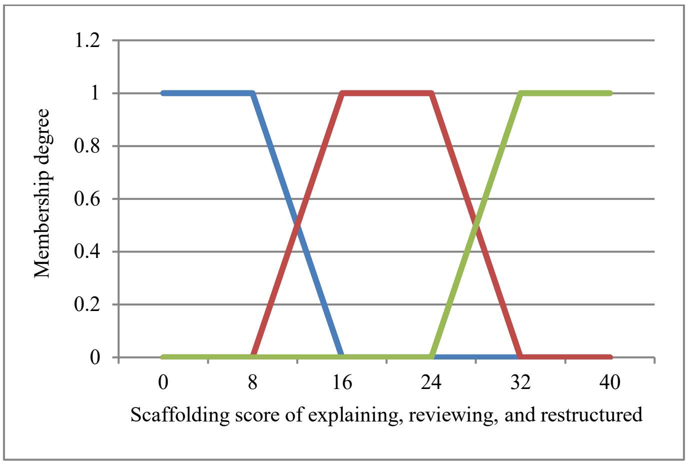

Figure 4. Model performance using a trapezoidal membership function for er input variable

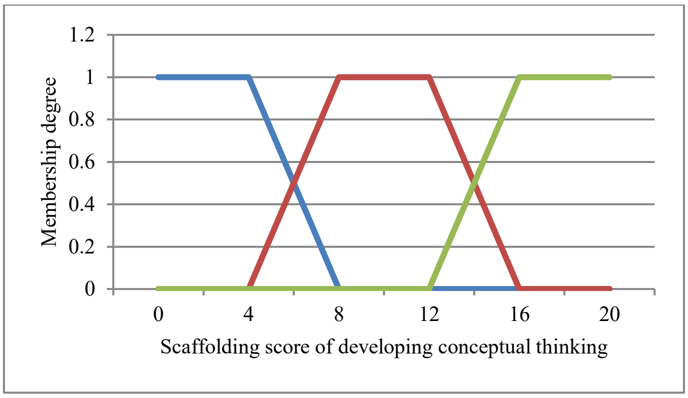

Figure 5. Model performance using a trapezoidal membership function for dc input variable

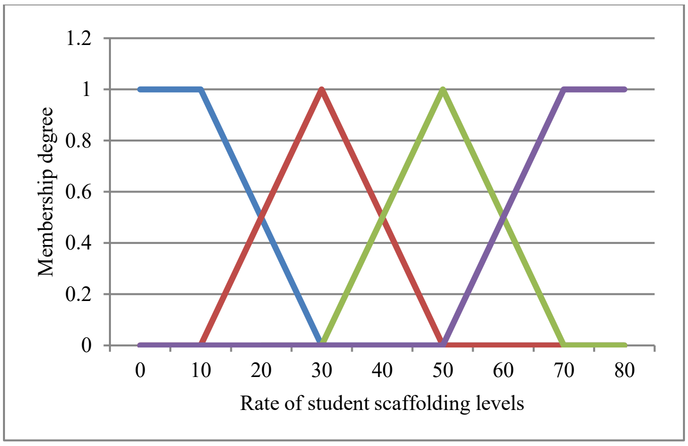

Figure 6. The membership functions for the output variable to representing scaffolding levels produced

Figures 3 to 5 illustrate the use of the inference system to generate the input for the fuzzy decision index. The equation used to determine the fuzzy membership value of the environmental provisions (ep) input set is divided into three fuzzy sets: weak (epR), moderate (epS), and strong (epT), as shown in (2) to (4). The value of x is the test results of geometry questions adjusted to Table 4. Before presenting the mathematical equations of the membership functions, the relationship between input variables and the corresponding fuzzy sets is summarized to improve clarity and readability. This summary allows students to understand the structure of the fuzzy model before the formal formulation.

$$\mu_{epR}(x) = \begin{cases} 1, & x < 4 \\ \frac{8-x}{4}, & 4 \leq x < 8 \\ 0, & x \geq 8 \end{cases} \tag{2}$$

$$\mu_{epS}(x) = \begin{cases} 0, & x < 4 \\ \frac{x-4}{4}, & 4 \leq x < 8 \\ 1, & 8 \leq x < 12 \\ \frac{16-x}{4}, & 12 \leq x < 16 \\ 0, & x \geq 16 \end{cases} \tag{3}$$

$$\mu_{epT}(x) = \begin{cases} 0, & x < 12 \\ \frac{x-12}{4}, & 12 \leq x < 16 \\ 1, & x \geq 16 \end{cases} \tag{4}$$

The equation used to calculate the fuzzy membership value of the input set for explaining, reviewing, and restructured (er) is divided into three fuzzy sets: weak (erR), moderate (erS), and strong (erT), as shown in (5) to (7). The value of x is the test results of geometry questions adjusted to Table 4.

$$\mu_{erR}(x) = \begin{cases} 1, & x < 8 \\ \frac{16-x}{8}, & 8 \leq x < 16 \\ 0, & x \geq 16 \end{cases} \tag{5}$$

$$\mu_{erS}(x) = \begin{cases} 0, & x < 8 \\ \frac{x-8}{8}, & 8 \leq x < 16 \\ 1, & 16 \leq x < 24 \\ \frac{32-x}{8}, & 24 \leq x < 32 \\ 0, & x \geq 32 \end{cases} \tag{6}$$

$$\mu_{erT}(x) = \begin{cases} 0, & x < 24 \\ \frac{x-24}{8}, & 24 \leq x < 32 \\ 1, & x \geq 32 \end{cases} \tag{7}$$

The equation to determine the fuzzy membership value for developing conceptual thinking (dc) is split into weak (dcR), moderate (dcS), and strong (dcT) fuzzy values, as shown in (8) to (10). The value of x is the test results of geometry questions adjusted to Table 4.

$$\mu_{dcR}(x) = \begin{cases} 1, & x < 4 \\ \frac{8-x}{4}, & 4 \leq x < 8 \\ 0, & x \geq 8 \end{cases} \tag{8}$$

$$\mu_{dcS}(x) = \begin{cases} 0, & x < 4 \\ \frac{x-4}{4}, & 4 \leq x < 8 \\ 1, & 8 \leq x < 12 \\ \frac{16-x}{4}, & 12 \leq x < 16 \\ 0, & x \geq 16 \end{cases} \tag{9}$$

$$\mu_{dcT}(x) = \begin{cases} 0, & x < 12 \\ \frac{x-12}{4}, & 12 \leq x < 16 \\ 1, & x \geq 16 \end{cases} \tag{10}$$

Table 4. Definition of the membership degree for the input variable related to scaffolding components

Input variable	Code	Fuzzy sets	Domain
Environmental provisions	ep	Weak, moderate, strong	[0–20]
Explaining, reviewing, restructuring	er	Weak, moderate, strong	[0–40]
Developing conceptual thinking	dc	Weak, moderate, strong	[0–20]
Output	level	Beginner, advanced, independent, high independence	[0–80]

3.2. Developing fuzzy rule base

The fuzzy rule base was created through expert judgment, involving three mathematics education specialists with doctoral degrees and over five years of experience in teaching geometry and instructional design. The experts independently proposed initial rule sets based on common student errors and scaffolding strategies in line with Pólya's problem-solving stages. A consensus process was conducted through focused discussions to refine and validate rules, ensuring consistency and pedagogical relevance. This expert-based validation process enhances the credibility and rigor of the proposed inference rules.

The 'rule base' is the most essential part of the FIS model. These rules are more straightforward to formulate using language rather than numbers and usually appear as 'IF-THEN' statements with fuzzy conditional logic. IF-THEN rules are composed of two parts: the antecedent, which represents the inputs, and the consequent, which indicates the result [39]. Activating all rules that have a true premise helps form the fuzzy conclusion set. The extent to which each rule activates depends on how closely the antecedent matches the input. This approximate matching forms the foundation for interpolating between potential input states and reduces the number of rules required to specify the input-output relationship. IF-THEN rules are essential because they enable the encoding of human expertise and knowledge via fuzzy rules. Different methods, including expert knowledge, expertise, and fuzzy models, can be employed to create fuzzy rules.

In this step, a set of fuzzy linguistic rules is established based on expert knowledge to develop the ranking model. The rules are established based on the decision maker's preferences to ensure suitable recommendations for scaffolding levels. The additional requirement for determining the scaffolding level is the criterion of a minimum component at a moderate level. By understanding the requirement, students can more easily receive appropriate learning support based on the defined scaffolding components and relevant levels. The FIS system, according to the requirements, is detailed in Table 5.

Table 5. The inference rule base of the Mamdani FIS model used to define scaffolding levels

Rule no		IF		THEN
	Ep	er	dc	level
1	Weak	Weak	Weak	Beginner
2	Weak	Weak	Moderate	Beginner
3	Weak	Weak	Strong	Beginner
4	Weak	Moderate	Weak	Beginner
5	Weak	Moderate	Moderate	Advance
6	Weak	Moderate	Strong	Advance
7	Weak	Strong	Weak	Beginner
8	Weak	Strong	Moderate	Advance
9	Weak	Strong	Strong	Independent
10	Moderate	Weak	Weak	Beginner
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
27	Strong	Strong	Strong	High independence

3.3. Defuzzification

Once the FIS process concludes, the defuzzification step transforms the output into precise, numerical values. Various techniques in fuzzy logic, like the centroid, height, and mean-max methods, can be used for defuzzification [40]. This problem employs the centroid method to derive a precise output value from the fuzzy output set produced during the previous inference process. Therefore, the value of z is determined using the centroid method.

The output for assessing scaffolding levels includes four fuzzy sets: beginner, advanced, independent, and high independence level. The fuzzy decision index is calculated by recalculating the decision level percentage based on the obtained crisp decision value. The constructed inference system is used to obtain the fuzzy decision index output as (11).

$$\mu_{LA}(z) = \begin{cases} 1, & z < 10 \\ \dfrac{30 - z}{20}, & 10 \le z < 30 \\ 0, & z \ge 30 \end{cases}$$

$$\mu_{LL}(x) = \begin{cases} 0, z < 10 \\ \dfrac{z - 10}{20}, & 10 \le z < 30 \\ \dfrac{50 - z}{20}, & 30 \le z < 50 \\ 0, & z \ge 50 \end{cases}$$

$$\mu_{LI}(x) = \begin{cases} 0, & z < 30 \\ \dfrac{z - 30}{20}, & 30 \le z < 50 \\ \dfrac{70 - z}{20}, & 50 \le z < 70 \\ 0, & z \ge 70 \end{cases}$$

$$\mu_{LKT}(x) = \begin{cases} 0,\ z < 50 \\ \frac{z-50}{20},\ 50 \le z < 70 \\ 1,\ z \ge 70 \end{cases} \quad (11)$$

3.4. Implementation of Mamdani FIS in Spreadsheet

The instrument consists of the interaction between the primary and several auxiliary spreadsheets. The primary spreadsheet provides a simulation for determining the level of scaffolding based on students' performance in solving geometry problems. Meanwhile, the auxiliary spreadsheets contain calculations related to the different components of scaffolding and corresponding levels, serving as data sources for the primary spreadsheet. This implementation was carried out on 30 additional cases, as shown in Table 6. The subject data is collected from personal information related to students' cognitive abilities, scaffolding elements, and problem-solving skills. This information is used as input to determine the appropriate level of scaffolding to support learning needs and enhance problem-solving abilities.

Table 6. Implementation of Mamdani FIS inference rule base for scaffolding level determination

Subject	Input			Output level
	ep	er	dc	
S1	13	16	00	Advance
S2	09	25	05	Advance
S3	09	30	06	Advance
S4	17	20	05	Advance
S5	04	35	14	Advance
S6	08	35	11	Advance
S7	15	35	14	Independent
S8	20	40	12	High independence
⋮	⋮	⋮	⋮	⋮
S27	06	19	03	Beginner
S28	05	31	07	Advance
S29	14	39	11	Independent
S30	16	30	13	Independent

3.5. Discussion

A decision support system for determining scaffolding levels is implemented using various computational methods. This system, which uses the Mamdani FIS method, is suitable for educational settings because it can manage uncertainty and ambiguity in student performance data. Fuzzy logic has been utilized in education to assist decision-making processes like scholarship selection, accreditation assessment, and program or major selection, showing the flexibility and relevance in complex human-centered domains [41]–[43]. The primary benefit of the Mamdani approach is its capacity to convert qualitative pedagogical insights into understandable fuzzy rules, which produce recommendations tailored to the context. However, the system does not replace teacher judgment but operationalizes the concept into a transparent decision-making framework.

Mamdani FIS was used to determine appropriate scaffolding levels based on students' error patterns in solving geometry problems. Mamdani FIS translated patterns of student errors across Pólya's problem solving stages into differentiated scaffolding levels. The system operates through a structured sequence of stages, including fuzzification of input variables, inference using expert-defined fuzzy rules, rule aggregation, and defuzzification to produce crisp scaffolding level outputs. This process allows diverse patterns of student errors in line with Pólya's problem-solving stages to be mapped into meaningful instructional support levels. Therefore, the system provides a transparent and interpretable mechanism for connecting student performance characteristics to scaffolding recommendations.

The main objective of integrating Mamdani decision support system is to support personalized learning by balancing instructional scaffolding with actual problem-solving needs. The system assists teachers in delivering support by classifying students into different scaffolding levels, enhancing students' independence and strategic thinking. Teachers can modify or expand the rule base to accommodate specific curriculum objectives, institutional requirements, or student characteristics. The scaffolding recommendations can be refined through additional rules or alternative parameter settings by leveraging the advantages of fuzzy logic, enabling the system to function as a customizable decision-support tool rather than a rigid evaluative mechanism. The spreadsheet-based implementation shows that the proposed system is practically feasible and accessible, allowing teachers and assessment teams to apply the concept without requiring advanced technical infrastructure. In real instructional settings, teachers make rapid instructional adjustments under cognitive load. The decision support system has the potential to alleviate the burden by externalizing diagnostic reasoning processes, enabling teachers to concentrate on pedagogical interaction rather than on subjective evaluative judgments [41]. However, classroom integration of the system is not inherently guaranteed. Teachers must receive pedagogical training to interpret scaffolding classifications as formative guidance rather than as prescriptive labels. In the absence of the training, the system may be misappropriated as a summative grading instrument, which contradicts the developmental and adaptive nature of scaffolding [21]. Therefore, successful adoption of the proposed system is dependent on professional development initiatives that explicitly connect system outputs to responsive and adaptive instructional practices [42].

The system outputs were descriptively compared with students' observed problem-solving behaviors and performance patterns during geometry instruction to examine the relevance of the generated scaffolding recommendations. The analysis shows a consistency between the recommended scaffolding levels and students' learning characteristics. This result supports the view that effective instructional support should be diagnosis-driven [43]. Students categorized at lower scaffolding levels tended to exhibit frequent conceptual misunderstandings and procedural errors, while those classified at higher independence levels reported more coherent solution strategies, effective self-monitoring, and reflective problem-solving behaviors. This correspondence suggests that the proposed Mamdani FIS produces recommendations pedagogically meaningful and balanced with students' actual learning needs.

The implementation of the system in real classroom environments presents opportunities and challenges. Teachers require targeted training to interpret system outputs and integrate the recommendations effectively into instructional decision-making. Moreover, student diversity in terms of learning styles, cultural backgrounds, and prior knowledge necessitates flexibility in applying scaffolding rules. The rule base should be treated as adaptable rather than universal. The flexibility of fuzzy logic enables contextual calibration, allowing students to refine membership functions or rules according to classroom characteristics [34]. This adaptability positions the system as a collaborative pedagogical instrument rather than a fixed intelligent tutor. Future extensions of the system incorporate adaptive mechanisms that respond dynamically to changes in student performance and extend the system's application to additional mathematical domains beyond geometry, such as algebra or calculus. The cross-domain extensions enable investigation into whether observed scaffolding patterns are domain-specific or reflect stable student characteristics, contributing to broader theoretical understandings of adaptive instruction. Mamdani FIS demonstrates potential and effectiveness in integrating decision support systems [44], [45]. The results show that the system has potential as a decision support tool for personalized scaffolding in mathematics education. The proposed approach contributes to more adaptive, transparent, and context-aware instructional support to improve students' problem-solving skills and learning results by connecting pedagogical theory with intelligent system design.

Despite the promising results, several limitations should be acknowledged. The fuzzy rule base depends on expert judgment, which may introduce subjectivity and limit the generalizability of the system across different instructional contexts or student populations. The current implementation is based on static input data and does not account for dynamic changes in learning trajectories. Addressing the limitations requires future studies that incorporate longitudinal data, adaptive rule-updating mechanisms, and empirical validation using quantitative learning outcome measures.

4. CONCLUSION

The decision support system for determining scaffolding can be adopted to make informed decisions about the appropriate level of support for students working on geometry problems. This system ensures that scaffolding is provided based on a careful analysis of students' abilities and errors, prioritizing the level of assistance needed. Teachers can provide support that addresses individual learning needs with the assistance of the system. Therefore, this study contributes significantly to helping students develop problem-solving skills by providing targeted scaffolding, enhancing learning outcomes. The system offers a more structured and systematic method for delivering personalized educational support.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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Susiswo	✓	✓			✓	✓	✓	✓	✓	✓		✓	✓	
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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

The authors state that they have no conflicts of interest. The manuscript was prepared independently, and no individual or institutional interests have influenced the design, data analysis, or outcomes. All processes strictly followed established ethical guidelines.

INFORMED CONSENT

Signed informed consent from individual students was waived because the study used pre-existing, historical academic records retrieved retrospectively from institutional archives. Student data were fully anonymized before analysis, and no direct interaction, intervention, or re-testing occurred with the participants to ensure privacy and ethical compliance.

ETHICAL APPROVAL

This research was carried out following the ethical guidelines set by the institutional committee. The study was exempt from formal ethical review and approval because the context used only fully anonymized secondary data on geometry assignment scores and included no risk to human participants. However, official permission to access and use the historical academic records was strictly obtained from the relevant institutional authorities.

DATA AVAILABILITY

The datasets used in this study, which include students' anonymized scores for geometry assignments, are not publicly accessible because of privacy restrictions related to student academic records. The data are available from the first author, [YIPP], upon reasonable request and with permission from the relevant institutional authorities.

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