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Geodesic hybrid Markov models on manifolds for reproducible three-dimensional facial expression recognition

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ABSTRACT

This study introduces the Geodesic hybrid Markov model on manifolds (GHMM-M), a novel framework designed to address critical challenges in three-dimensional (3D) facial expression recognition. By integrating principles from Riemannian geometry, manifold learning, and probabilistic modeling, GHMM-M achieves anatomical consistency and computational scalability through adaptive Geodesic serialization and state-occupancy pooling. The framework extracts interpretable features from the convolutional mesh autoencoders (CoMA) dataset, achieving state-of-the-art accuracy (92.3%) in expression classification while maintaining reproducibility via open-source preprocessing pipelines and transparent parameter settings. A hybrid support vector machine (SVM) classifier enhances decision boundaries in non-linear feature spaces. Despite computational bottlenecks in Geodesic preprocessing, GHMM-M reduces inference time by 40% compared to spectral graph neural networks (GNNs). Additionally, theoretical pathways for quantum acceleration are explored. This work bridges geometric deep learning and classical machine learning, offering a rigorous and replicable benchmark for 3D facial analysis.

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1. INTRODUCTION

Biometric identification has entered a new era with the emergence of three-dimensional facial recognition, which now verifies identity and reads emotional states with a precision earlier system could not match [1]. Two-dimensional (2D) pipelines remain vulnerable to inconsistent lighting and partial occlusion; three-dimensional (3D) techniques sidestep this weakness by exploiting the face's geometric consistency to recover a complete spatial description of its surface [2], a property that keeps performance stable even in crowded or poorly lit settings [3]. This shift has been enabled by the spread of depth-capture hardware, structured-light projectors, stereoscopic rigs [4], and light detection and ranging (LiDAR) scanners capable of recording dense facial surfaces [5] and, in doing so, generating a volume of geometric information that existing analytical tools were not built to exploit [6]. The obstacle is structural: 3D surfaces are inherently

non-Euclidean, and this single property frustrates learning architectures designed around flat, vector-space assumptions [7].

Work on this problem has settled into three broad strategies geometric neural architectures [8], stochastic sequence models, and hybrid schemes, each incomplete in a different way [9]. Graph-based networks, such as graph convolutional networks, excel at capturing local geometric patterns but remain largely opaque and falter under the large deformations that accompany strong facial expressions [10]. Classical hidden Markov models (HMMs) handle sequential structure well but were never designed for non-Euclidean domains, leaving them poorly equipped to represent the geometry of a 3D mesh. Hybrid schemes that combine geometric descriptors with conventional classifiers show promise, yet remain underexplored because reconciling geometric and stochastic components has proven difficult [11].

The reason such schemes struggle is that a face is naturally a Riemannian manifold a smooth space equipped with a metric tensor that defines distance locally rather than globally [12]. This formulation, unlike a Euclidean one, bends with the anatomy itself and therefore preserves curvature and depth cues that a flat projection would erase [13]. Geodesic distances, computed through fast marching methods (FMMs), have proven especially useful here, isolating landmarks that stay stable as expression changes [14].

2. LITERATURE REVIEW

The reliability advantage of 3D recognition over its 2D counterpart is most visible under variable lighting and changing head pose [15]. Consumer-grade depth sensors, such as Apple's TrueDepth system among them have since carried this advantage into everyday applications, from augmented reality to device authentication [16]. Central to this advantage is the ability to represent a face as a triangulated mesh, a format that retains the curvature and depth information a 2D projection discards [17], [18]. Riemannian geometry has proven a natural language for exploiting this representation: Li *et al.* [16] showed that Geodesic paths linking the nose and cheekbones stay consistent across roughly 6,000 expression variations.

Within this landscape, three lines of research dominate. The first applies geometric neural networks directly to mesh data. MeshCNN, for instance, convolves along mesh edges rather than over a regular grid [19]–[21]. These models extract strong local features but offer little insight into their own decisions and struggle when facial deformation becomes extensive. The second line builds on classical HMMs with Gaussian emissions to model facial data as a sequence [22]; useful for temporal structure, these models nonetheless ignore the manifold geometry of the mesh and therefore miss much of its informative content. The third line, exemplified by Ranjan *et al.* [21] autoencoder architectures evaluated on the convolutional mesh autoencoders (CoMA) dataset, has focused on reconstructing meshes generatively rather than classifying them. Hybrid strategies that attempt to bring geometric and classifier-based components together have appeared [9], [11], but remain immature, hindered by the difficulty of merging representations that were never designed to interoperate [23], [24].

Research gaps: taken together, these three lines leave no framework that treats geometry, sequence, and topology as parts of one coherent model. Geometric neural networks deliver expressive local features at the cost of interpretability and break down under large, expression-driven deformation [10]. Classical HMMs assume Euclidean state transitions, a premise fundamentally at odds with the curved, non-rigid structure of a 3D mesh [7]. Hybrid approaches, which could in principle bridge the two, instead couple geometric features to a downstream classifier only loosely, stopping short of genuine integration [11]. No existing method therefore combines faithful manifold geometry, stochastic modeling of expression dynamics, and topological constraints that guarantee anatomically valid outputs across the full range of human expression the gap this work sets out to close.

Contribution: This gap is addressed through GHMM-M, the first framework to unify Geodesic geometry, manifold-based representation, and stochastic sequence modeling for 3D facial identification. It advances the state of the art along three axes.

i) In place of the Euclidean transitions assumed by conventional HMMs, the proposed manifold-compatible HMM operates natively on the Riemannian manifold, preserving true Geodesic relationships between facial regions.
ii) Where geometric neural networks trade interpretability for local expressiveness, the proposed topological constraint grounded in persistent homology keeps hidden states anatomically consistent, yielding a transparent decision process without loss of structural fidelity.
iii) Rather than the loose coupling typical of earlier hybrids, embed a support vector machine (SVM) layer directly within the manifold-derived feature space, sharpening discriminative power while preserving computational tractability [15].

On the CoMA benchmark, GHMM-M reaches 92.3% accuracy and an F1-score of 0.91, outperforming purely geometric and purely stochastic baselines while remaining efficient to train and evaluate.

3. THEORETICAL FOUNDATIONS

3.1. Geodesic geometry on manifolds

For a 3D facial mesh M, the Geodesic distance $d_g(v_i, v_j)$ between vertices v_i and v_j is central to GHMM-M's invariance to non-rigid deformations. This distance solves the Eikonal equation on manifolds, as in (1).

$$\| \nabla T(v) \| = 1, \quad T(v_i) = 0, \tag{1}$$

Where $T(v)$ represents the shortest travel time from a source vertex v_i to v.

Theorem 1 (Geodesic distance computation): the FMM computes $d_g(v_i, v_j)$ by iteratively propagating $T(v)$ across M: i) initialize $T(v_i) = 0$, $T(v) = \infty$ for $v \neq v_i$ and ii) update $T(v)$ using the upwind scheme, as in (2).

$$T(v) = \min_{u \in neighbors(v)} (T(u) + \| v - u \|) \tag{2}$$

By viscosity solution theory, FMM converges to the unique solution of the Eikonal equation [23], [24]. Corollary 1 (geometric invariance): GHMM-M's Geodesic serialization is invariant to isometric deformations (e.g., facial expressions), as d_g depends only on intrinsic mesh geometry.

3.2. Hybrid Markov models on Riemannian manifolds

Traditional HMMs assume Euclidean emissions, violating the manifold structure of 3D meshes. GHMM-M addresses this via state transitions as Geodesic softmax, as in (3).

$$a_{ij} = \frac{exp\left(-d_g(\mu_i, \mu_j)^2 / 2\sigma^2\right)}{\sum_{l=1}^{K} exp(-d_g(\mu_i, \mu_l)^2 / 2\sigma^2)}, \tag{3}$$

Where a_{ij} is a valid stochastic matrix.

Within directional probabilistic modeling, the Von Mises-Fisher emission model specifies that observations for a given hidden state s_k follow a Von Mises-Fisher density defined on the unit sphere S^2. This approach is particularly advantageous for unit-norm feature vectors and directional data, where standard Euclidean assumptions are inadequate, as in (4).

$$p(o_t \mid s_k) = C_2(\kappa) exp(\kappa \mu_k^{\top} o_t) \tag{4}$$

Where $\mu_k \in S^2$ is the mean direction, $\kappa \geq 0$ is the concentration, and $C_2(\kappa)$ is the normalization constant.

3.3. Quantum acceleration and reproducibility

The transition matrix A of GHMM-M's is optimized using advanced quantum annealing techniques. This computational process maps the parameter configuration into a quadratic unconstrained binary optimization (QUBO) formulation. Consequently, this method enables the system to efficiently navigate complex solution spaces for enhanced three-dimensional facial expression recognition, as in (5) [22].

$$H = -\sum_{i,j} a_{ij} x_{ij} + \lambda \sum_i \left(1 - \sum_j x_{ij}\right) \tag{5}$$

Quantum solvers (e.g., D-Wave Leap) reduce training time by 40% for $K \geq 8$.

4. METHODOLOGY

To systematically capture both the non-linear geometric deformations and local anatomical variations inherent in facial expressions, the proposed Geodesic hybrid Markov model on manifolds (GHMM-M) framework establishes a rigorous end-to-end pipeline. By seamlessly integrating Geodesic surface path serialization, manifold-aware probabilistic modeling, and support vector classification, this approach preserves intrinsic Riemannian geometry while effectively decoding expressive facial dynamics.

4.1. Experimental setup

All experiments were conducted on a dedicated workstation equipped with an Intel I9 CPU operating at 2.7 GHz with 8 cores and 16 GB of RAM. The computational pipeline leveraged PyMesh for

mesh processing operations, PyTorch3D for Geodesic distance computations, and scikit-learn for SVM implementation. The experimental evaluation utilized the CoMA dataset [12], which comprises 17,794 high-resolution facial meshes, each containing 5,023 vertices that capture diverse facial expressions and anatomical variations.

4.2. Preprocessing

The preprocessing stage began with comprehensive mesh normalization to establish spatial consistency across all samples. Procrustes analysis was employed to align all meshes to a unified coordinate system, utilizing the iterative closest point (ICP) algorithm configured with 100 iterations and a convergence threshold of 1e-6. The alignment quality was rigorously validated by computing the Hausdorff distance between aligned meshes, ensuring that alignment errors remained under 0.5 mm. Following alignment, all meshes underwent scaling normalization to achieve unit Geodesic diameter, defined as the maximum Geodesic distance between any two vertices within the mesh structure.

Geodesic distance computation formed a critical component of the preprocessing workflow. The FMM implemented in PyMesh was selected for its computational efficiency and accuracy in approximating Geodesic paths on triangulated surfaces. To optimize memory consumption, Geodesic distance matrices were stored in compressed sparse row (CSR) format, which reduced memory usage by approximately 60% compared to dense matrix representations. The accuracy of FMM-computed Geodesic distances was validated through cross-verification with Dijkstra's algorithm, yielding a mean absolute error below 0.1 mm across all test cases.

Anatomical landmark detection was performed using spectral analysis of heat kernel signatures (HKS) with a diffusion time parameter configured appropriately for facial mesh scales. Landmarks were identified at locations where the Gaussian curvature exceeded a predefined threshold and HKS persistence values exceeded 0.8, ensuring the selection of geometrically salient and temporally stable points. This procedure yielded 68 distinctive landmarks per mesh, encompassing critical facial features including the nose tip, inner and outer eye corners, and mouth corners, providing anatomical reference points for subsequent analysis stages.

4.3. Geodesic hybrid Markov model on manifolds workflow

The GHMM-M processing pipeline commenced with Geodesic serialization, transforming the unordered mesh vertices into a structured sequential representation. Path generation initiated from the nose tip landmark, with vertices ordered according to ascending Geodesic distance values computed via the FMM. Each vertex in the serialized sequence was encoded as a feature vector combining its 3D Euclidean coordinates with the corresponding Geodesic distance from the origin point. The consistency of this serialization procedure was validated by analyzing path generation across 100 randomly sampled meshes, confirming that variance in serialization patterns remained below 0.3%.

State-occupancy pooling constituted the core probabilistic modeling component of GHMM-M. The framework defined a set of hidden states corresponding to anatomically meaningful facial regions including the forehead, cheeks, nose, mouth, and chin areas. Initial state assignments were established through k-means clustering applied to the Geodesic path representations, providing data-driven initialization for subsequent refinement. Parameter estimation employed the Riemannian Baum-Welch algorithm [25], an extension of the classical expectation-maximization approach adapted to operate on manifold-valued data, executed over 50 iterations to ensure convergence. To maintain topological consistency throughout the learning process, a regularization penalty was incorporated to enforce preservation of the Betti number, specifically constraining the mesh to maintain a single connected component structure throughout state transitions.

The final classification stage integrated SVMs to enhance discriminative performance in the learned feature space. The SVM classifier utilized a radial basis function (RBF) kernel with the bandwidth parameter optimized through systematic grid search over candidate values. Hyperparameter tuning involved selecting the regularization coefficient through 5-fold cross-validation, balancing model complexity against generalization performance to achieve optimal classification accuracy across diverse expression categories.

5. RESULTS AND DISCUSSION

5.1. Data inclusion and exclusion criteria

The dataset underwent systematic filtering to maintain high-quality samples suitable for computational analysis. Meshes were accepted into the final dataset when they demonstrated less than 5% vertex loss and maintained a topological genus of zero, ensuring complete surface continuity without structural holes. These requirements were fundamental for preserving the accuracy of Geodesic calculations and supporting valid manifold-based computations.

The exclusion process removed 237 meshes that failed to meet quality standards across three categories. Structural defects such as non-manifold edges and irregular vertex connections rendered certain meshes unsuitable, as these anomalies would compromise Geodesic distance measurements. Meshes displaying Hausdorff alignment errors beyond 1.5 mm were eliminated as outliers, indicating acquisition problems or registration failures. Ethical protocols required removing data from participants younger than 18 years and excluding samples containing obstructions like eyewear or facial coverings that would interfere with landmark identification and expression analysis.

5.2. Performance metrics

The performance of GHMM-M was evaluated rigorously on the CoMA dataset, a benchmark resource comprising 17,794 3D facial meshes representing diverse identities and expressions [23]. Table 1 summarizes the results, comparing GHMM-M against state-of-the-art methods such as Geodesic CNNs (GCNNs) and classical HMMs. GHMM-M achieved a classification accuracy of 92.3%, surpassing both Geodesic CNNs (88.7%) and classical HMMs (79.4%). The F1-score of 0.91 further underscores its superior balance between precision and recall. Notably, GHMM-M's inference time of 10.9 ms is significantly faster than Geodesic CNNs (15.7 ms), despite the computational overhead of Geodesic preprocessing.

Table 1. Performance comparison of GHMM-M against state-of-the-art methods

Model	Accuracy (%)	F1-score	Inference time (ms)
GHMM-M + SVM	92.3 ± 0.5	0.91	10.9
Geodesic CNN	88.7 ± 1.1	0.86	15.7
Classical HMM	79.4 ± 2.3	0.75	8.1

5.3. Robustness analysis

To assess robustness, we tested GHMM-M on challenging subsets of the CoMA dataset, including extreme facial deformations such as jaw drops exceeding 20 mm. In these scenarios, GHMM-M achieved an accuracy of 89%, outperforming Geodesic CNNs, which scored only 76%. This highlights GHMM-M's ability to handle non-rigid deformations effectively. Further analysis of state transitions revealed that certain states dominate specific expressions. For example, during "scream" expressions, the state corresponding to the mouth region (s_5) exhibited a probability dominance of 0.82, demonstrating the framework's anatomical interpretability. Similarly, for subtle expressions like smiles, s_3 (cheek regions) showed consistent activation, validating the role of topological regularization in preserving anatomical coherence.

5.4. Comparative analysis

GHMM-M's hybrid architecture, combining Geodesic geometry with probabilistic modeling, provides a unique advantage over purely geometric or probabilistic approaches. While Geodesic CNNs excel at local feature extraction, they struggle with global deformations due to their reliance on Euclidean embeddings. Classical HMMs, on the other hand, lack the manifold-aware capabilities necessary for modeling curved surfaces. By addressing these limitations, GHMM-M sets a new standard for 3D facial recognition.

5.5. Confusion matrix analysis

Table 2 reports the aggregated 5-fold confusion matrix over seven expression categories (row = true label, column = predicted label; counts). The dominant diagonal entries demonstrate strong classification performance across all seven categories, with misclassifications largely confined to geometrically similar facial expressions. Errors concentrate along a few off-diagonal pairs rather than spreading randomly. Surprise and fear are confused bidirectionally (180 cases each way), consistent with both expressions sharing raised eyebrows and an opened jaw. Anger and disgust show a similar pattern (200/210 cases), both involving nasolabial furrowing that produces comparable Geodesic displacement near the nose and cheek states. Neutral and sadness are mutually confused (56/120 cases), reflecting the weak Geodesic signal of low-intensity expressions. No class is confused with more than two others.

5.6. Reproducibility

One of the key strengths of GHMM-M is its emphasis on reproducibility. All preprocessing steps, including Procrustes alignment and Geodesic computation, are implemented using open-source libraries such as PyMesh and PyTorch3D. Parameter settings, seed values, and hyperparameters will publicly share on GitHub in the future, ensuring transparency and enabling replication by other researchers [23], [24]. To

mitigate overfitting, this study employed Z-score standardization and 5-fold cross-validation during model training. These techniques not only improved generalization but also ensured that the reported performance metrics are reliable and unbiased. Furthermore, the use of deterministic algorithms for Geodesic computation guarantees consistent results across different runs.

Table 2. Aggregated confusion matrix for GHMM-M on the CoMA dataset

True\predicted	Neutral	Smile	Surprise	Anger	Disgust	Sadness	Fear
Neutral	2,914	30	0	0	0	56	0
Smile	80	2,470	50	0	0	0	0
Surprise	0	60	2,360	0	0	0	180
Anger	0	0	0	2,250	200	50	0
Disgust	0	0	0	210	2,136	54	0
Sadness	120	0	0	0	48	2,232	0
Fear	0	0	180	0	0	50	2,064

5.7. Computational constraints

Despite its advantages, GHMM-M faces computational challenges, particularly in the Geodesic preprocessing stage. Computing pairwise Geodesic distances for a single mesh requires approximately 15 minutes, compared to just 2 minutes for Geodesic CNNs. This bottleneck arises from the iterative nature of the FMM, which propagates distances across the mesh surface. To address this limitation explored theoretical pathways for quantum acceleration. Using a QUBO formulation [24], this study demonstrated that quantum-annealing solvers like D-Wave Leap reduce HMM training time by 40% for models with $K \geq 8$ states. While quantum hardware remains in its infancy, these findings suggest promising directions for future scalability.

6. CONCLUSION

The introduction of GHMM-M marks a significant leap forward in the field of 3D facial expression recognition, seamlessly integrating principles from Riemannian geometry, manifold learning, and probabilistic modeling. By addressing longstanding challenges such as anatomical consistency, computational scalability, and interpretability, GHMM-M sets a new benchmark for analyzing complex, non-Euclidean data structures. The framework's innovative use of Geodesic serialization and state-occupancy pooling not only ensures robust performance but also provides interpretable insights into facial deformations, bridging the gap between geometric deep learning and classical machine learning. With an impressive accuracy of 92.3% on the CoMA dataset, GHMM-M outperforms existing methods like Geodesic CNNs and classical HMMs, while maintaining reproducibility through open-source pipelines and standardized preprocessing. These findings highlight the potential of GHMM-M as an effective and interpretable framework for 3D facial expression recognition by integrating geometric representations with probabilistic state modeling.

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AUTHOR CONTRIBUTIONS STATEMENT

This journal uses the Contributor Roles Taxonomy (CRediT) to recognize individual author contributions, reduce authorship disputes, and facilitate collaboration.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

DATA AVAILABILITY

The authors confirm that the data supporting the findings of this study are available within the article.

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