Fuzzy logic based sliding surface adjustment of second-order sliding mode controllers

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ABSTRACT

This research work designs a variant of second-order sliding mode control scheme, making use of varying sliding surface inferred using a fuzzy inference system. The varying sliding surface is an effective strategy to improve controller performance. A surface with a relative degree of two is first built by accounting for the uncertainties and perturbances of the system. Thereafter, in order to enhance the dynamics of the system being controlled, a varying sliding surface based on a straightforward double input-single output fuzzy logic inference architecture is proposed. The controller ensures system's reaching conditions, and also the stability and robustness. The designed control scheme is studied in comparison with a sliding mode controller of second order having a constant surface of sliding using Simulink based simulation for a nonlinear system. The comparison shows that the proposed strategy exhibits an improved dynamic performance than the conventional sliding mode control of second order having a constant surface of sliding.

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1. INTRODUCTION

The sliding mode controller (SMC) strategy has proved itself as a very established control strategy that has been extensively and successfully utilized in the control of dynamic systems with model and parametric uncertainties, and unanticipated disturbances. This popularity of the sliding mode control scheme arises from its desirable properties, which include its opposition to external shocks and disturbances, variations in model parameters, and system and model uncertainties. Furthermore, SMC algorithm is a clear, straightforward and robust one. The steps involved in the basic SMC design are designing a suitable surface of sliding, and the enforcing the mode of sliding. The basic SMC scheme utilizes either a unit controller or a relay controllers [1]. The SMC have proved to be effective for stabilizing uncertain non-linear systems [2]–[4]. One basic problem with this control scheme is that switching and temporal delays in system dynamics hinder the system profile from reaching the ideal sliding mode, which results in a phenomenon called chattering, which is nothing but an oscillation of high frequency [5]–[11]. Additionally, while the system profile is in the reaching mode, the basic SMC having a constant surface of sliding suffers from the restraint of difficulty in managing tracking error, making the system more vulnerable to parameter changes [1].

The second-order sliding mode controller (SOSMC) strategy is a modified version of SMC using higher-order sliding modes to ensure tracking performance and robustness even when the uncertainties and unanticipated disturbances are present in the system [12]–[15]. In SOSMC strategy, the control regime is
such that it forces the system states onto a sliding surface, which is a manifold of reduced dimensionality, such that the system dynamics are constrained to evolve along the surface. The controllers which are of SOSMC type have a performance which depends very much on the selection of the surface of sliding. Moreover, SOSMC having a constant surface of sliding are more susceptible to parameter changes when it is in its mode of reaching the surface, this sensitivity can be taken care of by shortening the time to reach the surface.

Fuzzy logic is a very popular soft computing tool that has been extensively used in various industrial applications because of its inherent ability to incorporate uncertainties and approximations [16], [17]. It is particularly an effective tool for control problems for which an exact model is not available. The algorithm of a fuzzy adaptive SOSMC type intended for controlling a particular type of nonlinear systems was addressed in [18]. To reduce chattering, a fuzzy SMC algorithm has been developed using SMC technique combined with fuzzy logic in [19]. An innovative adaptive super-twisting SMC using fuzzy inference is used in [20], which regulates dynamic uncertain systems. Hence, fuzzy inference system has been widely used in combination with SMC schemes for combining the advantages offered by two popular schemes.

The major disadvantage of the basic SOSMC having a constant surface of sliding is the high dependence of the system performance on the choice of the surface, even though SOSMC scheme for the control of uncertain systems reduces the phenomenon of chattering inherently present in the basic first SMC. Moreover, they guarantee better accuracy in the presence of system imperfections, perturbances and uncertainties. Finding an ideal surface of sliding is a very difficult, time-consuming and almost an impossible task. Hence, utilizing a varying sliding surface becomes an ideal choice for improving the performance of SOSMC controllers. But, there is no formal and strict rule in the design of varying sliding surface [21]–[26]. Hence, utilizing the fuzzy inference system is the best choice for the design of varying the surface.

This paper designs a fuzzy inference model of double input-single output type for the design of varying sliding surface of the SOSMC controllers. This scheme has the advantage that the sliding surface can be modified online in accordance with the values of the sliding variables to achieve the performance specifications. Additionally, the sliding surface can rotate either clockwise or counter-clockwise so as to enhance the dynamics of the system being controlled. As the sliding surface rotation is inferred utilizing a double input-single output fuzzy inference model, the developed scheme is a relatively simple methodology with quick computation time. Results from computer simulations show that the proposed SOSMC scheme outperforms the conventional SOSMC scheme having a constant surface of sliding, in terms of enhancing the dynamic performance.

2. METHOD

The proposed SOSMC technique is the modification of the SOSMC algorithm presented in [18]. The control design process has two steps. A novel SOSMC controller is initially developed step-by-step in the first stage utilizing a modified version of the SOSMC technique, and a thorough mathematical analysis is also performed. The second phase presents a comprehensive simulation strategy for the suggested SOSMC algorithm.

2.1. Brief description of second-order sliding mode controller

A non-linear uncertain system represented by (1) is considered here.

\[ \dot{x} = f_1(x, t) + g_1(x, t)U, \quad s = s(x, t) \]  

(1)

where \( x \in \mathbb{R}^n \), \( x \) representing the states of the system, and \( U \in \mathbb{R} \), \( U \) representing the input, \( f_1(x, t) \) and \( g_1(x, t) \) represent the functions, which are smooth, the output being the variable of sliding, \( s \in \mathbb{R} \). The variables of sliding \( s \) and \( \dot{s} \) are considered that they are known. If \( s \) has a degree of relativeness \( r = 2 \) with reference to the controller \( U \), then:

\[ \ddot{s} = a(t, x) + b(x, t)U \]  

(2)

where \( a(x, t) = \dot{s} \), under \( U = 0 \) and \( b(x, t) = \frac{\partial \dot{s}}{\partial U} \). The SOSMC operates under the modes: \( U=1 \) or \( U=-1 \). The switch \( \mu \) can be identified as in (3) [26]:

\[ \mu = \frac{1}{2} \left( 1 + \text{sign}(U) \right) \]  

(3)

However, it is very clear from (3) that the signum function results in a switching frequency which is infinite, whenever the variables of sliding is such that \( |\dot{s}|^2 + \beta \dot{s} = 0 \). In (3) gives a very high frequency of switching
for controller. Therefore, this approach cannot be directly implemented for the system being controlled. The frequency of switching can be limited to be within a range by utilizing the hysteresis modulation given by (4) within the region the region \( \Omega = -1 < [\dot{s}]^2 + \beta s < 1. \)

\[
sat([\dot{s}]^2 + \beta_1 s) = \begin{cases} 
-1, & \text{for } [\dot{s}]^2 + \beta_1 s < -1 \\
[\dot{s}]^2 + \beta_1 s, & \text{for } -1 < [\dot{s}]^2 + \beta_1 s < 1 \\
1, & \text{for } [\dot{s}]^2 + \beta_1 s > 1 
\end{cases} \tag{4}
\]

The action of switching does not happen in region \( \Omega \) after this alteration. This alteration results in reducing SOSMC’s frequency of switching from infinity, and error in the output is definitely converging to \( |\dot{s}|^2 + \beta_1 s | < 1 \), i.e., the variables of sliding is definitely converging to \(|[\dot{s}]^2 + \beta_1 s| < 1\). if \( V(s) = \frac{1}{2} s^2 \), then it is explicit that \( \dot{s}|s| < 1 - \beta_1 s \). This computation yields \( \dot{V}(s) \leq \frac{-\beta_1 s^2 + |s|}{|s|} \), suggesting that the variables of sliding will definitely converge to \( s: |s| \leq \frac{1}{\beta_1} \).

2.2. Varying surface of sliding for second-order sliding mode controller

An issue with a SOSMC controller having a constant surface of sliding is that the system is more susceptible to parameter changes when it is in reaching mode regime, the sensitivity can be decreased by shortening the duration of the reaching mode regime. Furthermore, it is difficult and time-consuming to determine the appropriate and the best sliding surface. A sliding surface design strategy that effectively improves the performance is to choose a varying surface of sliding as opposed to fixed ones, the variation is based on the variables \( s \) and \( \dot{s} \). Designing an SOSMC controller \( U \) is now necessary for the output \( x_1 \) to follow the desired value \( x_{1id} \). The modified SOSMC having a varying surface of sliding for system (2) is designed as given in (5).

\[
U = -s\text{ign}([\dot{s}]^2 + \beta_1(s,\dot{s}) s)
\tag{5}
\]

with an appropriately tuned value for \( \beta_1 (s, \dot{s}) \).

2.3. Fuzzy logic based sliding surface adjustment of second-order sliding mode controllers

The algorithm is a controller of SOSMC type with a new sliding surface given by \([\dot{s}]^2 + \beta_1(s,\dot{s}) s\). However, providing a clear formula to compute the parameter \( \beta_1 (s, \dot{s}) \) is challenging. The approximate rule for designing \( \beta_1 (s, \dot{s}) \) is derived by studying the dependence of the system response on the slope \( \beta_1 \). It is clear that the controller with maximum value of gradient \( \beta_1 \) exhibits error convergence at a faster rate, but at the cost of sacrificing the accuracy of tracking. However, if \( \beta_1 \) has a value which is of large magnitude, the states of the system will have an unacceptable overshoot and may lead to unacceptable performance. Hence, there always exists a reciprocation between tracking performance and error convergence. This can be accounted by moving the sliding surface as illustrated in Figure 1.

![Figure 1. Sliding surface movement](image)

To ensure stability, the sliding surface gradient must always be positive. The variation of the surface can be accomodated by inferring online the value of the sliding surface gradient based on two quantities-sliding variable \( s \) and its time derivative \( \dot{s} \). The link between the error variables and the gradient of the sliding surface cannot be exactly modelled. Therefore, a double input-single output fuzzy inference model is developed based on the approximation rules base.

The variables \( s \) and \( \dot{s} \) are scaled by suitable input scaling factors to bring them in the range [-1,1] and then applied as inputs to a double input-single output fuzzy inference model. The output of the fuzzy
inference model is then defuzzified to give a value in the range [0,1], and then subjected to a suitable output scaling factor to give the output, which is surface gradient $\beta_1(s,\dot{s})$. Figure 2 shows the fuzzy sets associated with the input variable $s$ and Figure 3 shows the ones associated with the input variable $\dot{s}$. The linguistic variables of $s$ and $\dot{s}$ are “negative big (NB),” “negative medium (NM),” “negative small (NS),” “zero (ZE),” “positive small (PS),” “positive medium (PM)” and “positive big (PB).” The fuzzy sets of the output variable $\beta_1(s,\dot{s})$ are shown in Figure 4, linguistic variables being “very very small (VVS),” “very small (VS),” “small (S),” “medium (M),” “big (B),” “very big (VB)” and “very very big (VVB).” The inference system has the rule base as presented in Table 1. Mamdani based fuzzy inference is proposed. Defuzzification can be accomplished using the centroid approach. The control strategy can now be illustrated as seen in Figure 5.

<table>
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<tr>
<th>$s$</th>
<th>$\dot{s}$</th>
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3. RESULTS AND DISCUSSION

The SOSMC scheme having a varying surface of sliding based on double input-single output fuzzy inference model is compared with SOSMC having a constant surface of sliding proposed by Mei et al. [18] for a nonlinear system. Figures 6 to 13 show the Simulink based simulation results for the SOSMC having a varying surface of sliding and the SOSMC having a constant surface of sliding. Figure 6 shows the system responses for the proposed SOSMC having a varying surface of sliding, and the SOSMC having a constant surface of sliding for different values of $\beta_1$. The proposed SOSMC scheme responds faster than the SOSMC having a constant surface of sliding.

The proposed SOSMC and SOSMC with $\beta_1=10$ and $\beta_1=2.5$ have rise times of respectively 0.028 sec, 0.238 sec, and 0.475 sec, and settling times of respectively 0.033 sec, 0.285 sec, and 0.565 sec. The system with SOSMC having a varying surface of sliding takes 0.039 sec for the response to move to the peak, and it is respectively 0.347 sec and 0.693 sec for the conventional SOSMC with $\beta_1=10$ and $\beta_1=2.5$. All the controllers ensure zero steady-state error and zero overshoot. Figure 7 represents the sliding variables. It is explicit that SOSMC having a varying surface of sliding achieves a faster response than the conventional SOSMC scheme by exerting significantly higher control effort during the first phase as shown in Figure 9. Figure 10 shows that the proposed SOSMC scheme exhibits a faster error convergence. The proposed SOSMC is faster throughout the response, as indicated by the integrated absolute error (IAE) and integral time absolute error (ITAE) curves shown in Figures 11 and 12 respectively. The IAE indices for the proposed SOSMC and the conventional SOSMC with $\beta_1=10$, and the conventional SOSMC with $\beta_1=2.5$ are respectively 0.004, 0.035, and 0.069, and ITAE values are respectively 0.012, 0.104, and 0.208, confirming that the proposed scheme ensures that the system responds faster than with SOSMC having a constant surface of sliding. Figure 13 shows the responses of the proposed system for various initial conditions, which show that the proposed SOSMC scheme exhibits similar response for various initial conditions, thereby exhibiting robust performance. The performance metrics for the responses are summarised in Table 2. According to simulation results, the proposed SOSMC scheme responds faster than the conventional SOSMC scheme and improves the dynamic response of the system. However, the controller does not sacrifice the desirable properties of robust control, stability and accuracy.
Figure 8. Rate of change of sliding variable under SOSMC having a varying surface of sliding and SOSMC with $\beta_1 = 10$ and $\beta_1 = 2.5$

Figure 9. Control inputs under SOSMC having a varying surface of sliding and SOSMC with $\beta_1 = 10$ and $\beta_1 = 2.5$

Figure 10. Error convergence of $x_1$ under SOSMC having a varying surface of sliding and SOSMC with $\beta_1 = 10$ and $\beta_1 = 2.5$

Figure 11. IAE of the response under SOSMC having a varying surface of sliding and SOSMC with $\beta_1 = 10$ and $\beta_1 = 2.5$

Figure 12. ITAE of the response under SOSMC having a varying surface of sliding and SOSMC with $\beta_1 = 10$ and $\beta_1 = 2.5$

Figure 13. Responses of the SOSMC having a varying surface of sliding for different initial conditions
4. CONCLUSION

This work proposes a novel fuzzy inference system based second order sliding mode control scheme. Firstly, it is demonstrated that the dynamic response of the SOSMC can be altered by rotating the surface of sliding using a fuzzy inference system. The results from Simulink based simulations of the control of a non-linear uncertain dynamic system are used to verify the effectiveness of the proposed control scheme. The proposed control approach is studied in comparison with a conventional SOSMC having a constant surface of sliding. The results prove that the proposed SOSMC having a fuzzy inference system based varying surface of sliding has a faster dynamic response, which can be read as lowering the reaching mode time and hence enhancing the dynamics. The improved dynamic performance was ensured without any effect on stability, tracking accuracy and robustness. In addition, the proposed control scheme is simple, requires less computing time and easy to implement.

REFERENCES


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<th>Conventional SOSMC with β1=2.5</th>
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<td>Rise time (sec)</td>
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<td>0.238</td>
<td>0.475</td>
</tr>
<tr>
<td>Settling time (sec)</td>
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<td>0.285</td>
<td>0.565</td>
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<tr>
<td>Peak time (sec)</td>
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<td>0.693</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0.012</td>
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